Minimum Pressure Envelope Cavitation Analysis Using Two-Dimensional Panel Method

by

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ABSTRACT

An analysis tool for calculating minimum pressure envelopes was developed using XFOIL. This thesis presents MATLAB® executables that interface with a modified version of XFOIL for determining the minimum pressure of a foil operating in an inviscid fluid. The code creates minimum pressure envelopes, similar to those published by Brockett (1965). XFOIL, developed by Mark Drela in 1986, is a design system for Low Reynolds Number Airfoils that combines the speed and accuracy of high-order panel methods with fully-coupled viscous/inviscid interaction. XFOIL was altered such that it reads in command line arguments that provide operating instructions, rather than by an operator interation via menu options. In addition, all screen output and plotting functions were removed. These modifications removed XFOIL's user interface, and created a "black box" version of XFOIL that would perform the desired calculations and write the output to a file. These modifications allow rapid execution and interface by an external program, such as MATLAB®. In addition, XFOIL's algorithms provide a significant improvement in the accuracy of minimum pressure prediction over the method published by Brockett.

Development of the modified XFOIL and MATLAB® interface contained in this thesis is intended for future interface with *Open-source Propeller Design and Analysis Program* (OpenProp). OpenProp is an open source MATLAB®-based suite of propeller design tools. Currently, OpenProp performs parametric analysis and single propeller design, but does not perform cavitation analysis. Minimum pressure envelopes provide the propeller designer information about operating conditions encountered by propellers. The code developed in this thesis allows the designer to rapidly assess cavitation conditions while in the design phase, and make modifications to propeller blade design in order to optimize cavitation performance. A methodology for design is discussed outlining future integration with OpenProp.

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1 Introduction

The study of propeller cavitation and its inception is an important aspect of propeller design. In order to accurately predict cavitation inception, it is necessary to be able to determine the actual pressure distribution in the fluid. By comparing the pressure coefficient to the local cavitation number, an estimate of the local cavitation conditions may be made. This is often accomplished by determining the fluid velocity distribution in the fluid, and then using the velocity to calculate local pressure conditions. Specifically, the pressure distribution is desired along the upper and lower surfaces of the foil in order to determine lift, drag, moment, and cavitation inception.

An early approach to this problem was to assume that the working fluid was inviscid. This assumption allowed the use of potential flow theory to calculate velocity as a function of location within the fluid. However, this method was limited to very simple shapes, such as a two-dimensional cylinder. Potential theory lacked the ability to directly calculate the fluid velocities around complex geometries such as foil surfaces.

Conformal mapping provided a method by which the exact velocity distribution could be calculated for certain types of foil shapes. However, exact conformal transformations for all foil shapes are not possible. Although various transformations have been introduced, this project uses the Karman-Trefftz foil as a comparison for numerical approaches. To further extend the use of conformal transformations, numerical approaches were developed to approximate the mapping function for foils of arbitrary shape.

An improved approach to obtain an accurate estimate of the actual pressure distribution on two-dimensional foils of arbitrary shape was developed by Brockett [1]. Brockett's work was based on the work of Moriya [2] which is an approximate conformal transformation of the circle to an airfoil profile and gives equations for the velocity distribution. Brockett published a FORTRAN computer program that would

accept foil ordinate input, in various formats, and calculate the velocity and pressure distributions at a specified angle of attack or lift coefficient.

The purpose of this report is to present a modified approach to the work of Brockett, and present an improved method for computing the minimum pressure envelopes of a foil. As a reference, Brockett's method of calculating the minimum pressure distribution was programmed into MATLAB. Sample input provided by Brockett was used to verify accuracy of the MATLAB version of Brockett's work.

A modified approach using the two-dimensional panel method of XFOIL is presented. Rather than using the approximate conformal transformation method presented by Brockett, a modified version of XFOIL was used to perform the calculations for pressure distribution. The modified XFOIL executable removed all interactive user interfaces, including menu driven options and interactive screen output. In addition, desired output is saved as a text file, rather than plotting to the screen. The development of the modified XFOIL executable allows the use of an external program, in this case MATLAB, to call XFOIL to perform the desired calculations. The results are then saved as a text file, and may be read in by MATLAB, which conducts the desired analysis and output.

A Karman-Trefftz foil was used as a reference for comparison. The analytic solution for the pressure distribution on the foil was used as the baseline to which the numerical methods were compared. The method presented using XFOIL to conduct calculations is nearly indistinguishable from the analytic solution, a significant improvement over the Brockett method that underestimated the minimum pressure by 28% at a high angle of attack (10°).

The intention of this project was to develop an improved method for computing minimum pressure envelopes for an arbitrary foil shape. In addition, it was developed such that this method would be integrated into the *Open-source Propeller Design and Analysis Program* (OpenProp). OpenProp is an open source MATLAB®-based suite of propeller design tools. OpenProp currently performs

parametric analysis and single propeller design, but does not perform cavitation analysis. The development of the MATLAB code in this project would aid the designer in the rapid design of propellers by providing a quick method to predict cavitation performance of a propeller, and allow the analysis of cavitation performance early in the design process. Conceptual implementation will be discussed later in this report.

2 Conformal Transformations

2.1 History

Prior to the development of the computer, obtaining an accurate solution for the flow around a complex shape was a challenging task. The development of conformal transformations was therefore of great benefit, as it provided an analytic solution for the exact inviscid flow solution to a select number of foil shapes. This method was developed by Joukowski in 1914. Karman and Trefftz then introduced a more general mapping function, which was a special case of the Joukowski transformation. Theodorsen [3] then built upon this work and developed an approximate numerical technique for obtaining the mapping function of an arbitrary foil shape. These developments ultimately led to the work of Brockett, and his development of the design charts published in 1966[4].

2.2 Use of Conformal Transformations

Although a detailed explanation of conformal transformations is not warranted here, the motivation of this project deserves a brief description of the procedure of conformal transformations, in order to highlight the significant improvements of the work presented. The below derivation is an adaptation of reference [5]

Potential flow solution for a two-dimensional cylinder is easily described and understood. It consists of a source-sink dipole, oriented by the direction of the uniform stream. This produces streamlines that define the two-dimensional shape shown in Figure 1.

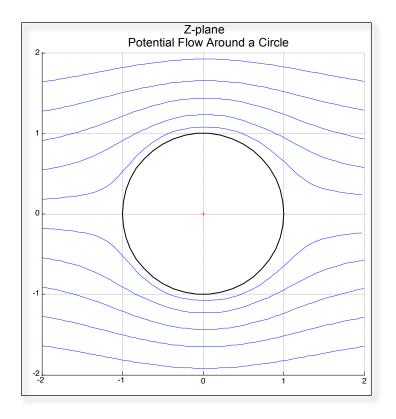


Figure 1: Potential Flow Solution for 2-D Cylinder

The Karman-Trefftz transformation maps a complex point z, where z = X + i*Y, from the Z-Plane to a point ζ using equation (2.1) below.

$$\xi = \frac{\lambda a \left[\left(z + a \right)^{\lambda} + \left(z - a \right)^{\lambda} \right]}{\left[\left(z + a \right)^{\lambda} - \left(z - a \right)^{\lambda} \right]}$$
 (2.1)

In order to evaluate the velocities in the ζ -Plane the derivative d ζ /dz must be evaluated using equations (2.2) and (2.3) below.

$$\frac{d\zeta}{dz} = \frac{4\lambda^{2}a^{2} \left[(z-a)^{\lambda-1} (z+a)^{\lambda-1} \right]}{\left[(z+a)^{\lambda} - (z-a)^{\lambda} \right]^{2}}$$
(2.2)

$$\left[u - iv\right]_{\xi} = \frac{\left[u - iv\right]_{\xi}}{\frac{d\xi}{dz}} \tag{2.3}$$

In order to map the circle into the ζ -plane, the Kaman-Trefftz method requires that the circle center be defined by x_C and y_C . Also, the trailing edge angle, τ , is defined in degrees and related by $\lambda = 2 - \tau/180$. Finally, a, which is the X-intercept, is set equal to unity. The foil in Figure 2 below was developed by defining $x_C = -0.1$ and $y_C = 0.15$, and $\tau = 10^\circ$. In addition, an angle of attack may be specified, however, in Figure 1 and Figure 2, the angle of attack is zero. Lastly, the circulation is set in order to meet the Kutta condition to ensure smooth flow leaving the trailing edge.

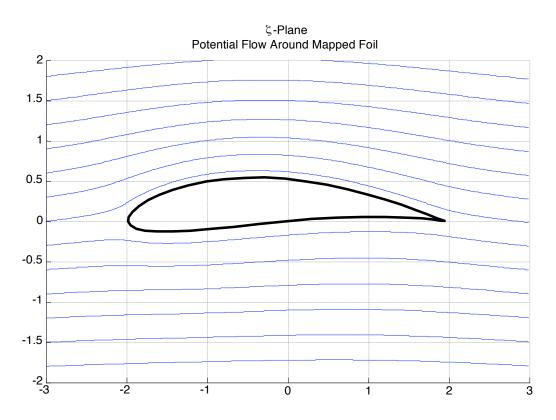


Figure 2: Karman-Trefftz Foil and Streamlines

To evaluate the velocity and calculate the pressure distribution for the ideal fluid flow over the foil surface, the velocities in the ζ -Plane were evaluated using Equation (2.3), and the pressure coefficient, Cp, was calculated using Equation (2.4), where q is the absolute velocity, and U is the free stream velocity.

$$C_{p} = \frac{p - p_{\infty}}{\frac{1}{2}\rho U^{2}} = 1 - \left(\frac{q}{U}\right)^{2}$$
 (2.4)

Since the Karman-Trefftz foil provides an analytic solution for the potential flow around the mapped foil, it provides an exact solution to which numerical methods may be compared. Figure 3 shows the analytic pressure distribution for the Karman-Trefftz foil shown in Figure 2.

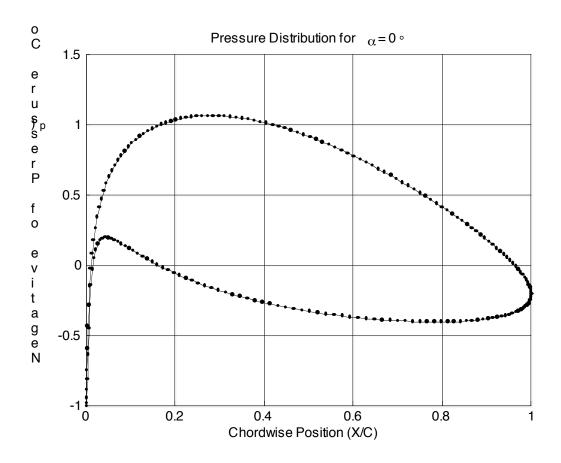


Figure 3: Karman-Trefftz Foil Pressure Distribution.

The analysis above is the result of the MATLAB script contained in Appendix A:

<u>Matlab Script for Conformal Transformation of Karman-Trefftz Foil.</u> This script also generates and exports data for further analysis.

3 Brockett's Analysis

3.1 Introduction to Brockett's Analysis

In reference [1], Brockett developed a computer code to evaluate the steady twodimensional pressure distribution on arbitrary foils, and presented the results. This code is based on an approximate potential theory suggested by Moriya which is empirically modified in a manner suggested by Pinkerton to give an arbitrary lift for a set incidence, while satisfying the Kutta condition. Interpolation functions for the ordinates were used to reduce the calculations to a straight-forward numerical procedure. Brockett presents a FORTRAN computer program, which, as part of this thesis, was rewritten as a MATLAB script in order to facilitate simple input and output. The original code was left unaltered as much as possible, such that the algorithms were left intact. The only significant changes are the method in which data was fed to the program, and the programming structure as required by MATLAB. Also, MATLAB allows variables to be defined by the user, or data can be read in by MATLAB, rather than using FORTRAN control card format specified by Brockett. This functionality greatly enhances the interface capability of the program by the user, and allows more rapid analysis and comparison of results. The code is presented in Appendix B: MATLAB Code of Brockett's Work, and a description of the main variables used in the program are included in Appendix C: Brockett.m Variable Descriptions.

3.2 User Input to MATLAB Version of Brockett Code

FORTRAN used formatted control cards, which allowed the user to input data to be processed by the program. This method of data has been superceded by either direct input by the user or digital data saved as files on the computer. The MATLAB code developed maintains as much original structure as possible, while allowing the operator to specify which data will be used as input, which is accomplished by the use of MATLAB script files. These files allow the user to specify the data to be processed for each of the formats required by Brockett's code. In reference [1], Brockett provides sample input and output of the original code. The first script file in Appendix D: Sample Input Scripts for Brockett.m., REQD IN.m, duplicates the

sample input presented by Brockett, and was used as a validation case for the MATLAB code. Results matched the values presented in Brockett's sample output on pages 70 – 72A of reference [1]. This format of this script file is required when inputting data at the points at the required offset locations as required by BROCKETT.m.

Also included in Appendix D: , are ARB_IN.m, KT_IN.m, and Brock_IN.m. The ARB_IN.m is a sample file for the format required to input foil ordinates at arbitrary locations. ARB_IN.m is written to accept input similar to that presented by Brockett in Figure 4b of reference [1]. The format is for input of foil ordinates at arbitrary stations. The format allows multiple angles of attack to be input, resulting in the calculation of pressure distribution at each angle of attack. This format accepts input, as specified by Brockett, that specifies airfoil ordinates in X, Y format. Data points are required to be entered with the same number of points on the upper and lower surface, starting with the trailing edge along the upper surface to the nose location, continuing along the lower surface to the trailing edge. KT_in.m is a script file used to import the data generated by the conformal transformation of Appendix A: , used to compute the pressure distribution predicted by Brockett. Brock_IN.m is used to input the geometry for the NACA 66, a = 0.8 (TMB Modified) foil used by Brockett in reference [4]

Brockett's FORTRAN program required very specific format for inputting data. Since the original code was maintained similar to the original structure, several operational variables must be specified for the code to function properly. In addition, input data must be carefully structured in the proper format to be processed correctly. The script files of Appendix D: were used to accomplish the variable definitions required by <code>Brockett.m</code>.

3.3 Output from Brockett.m

To validate that the MATLAB version of Brockett's code was accurate, a test run was conducted that replicated the sample case included in reference [1]. The REQD IN.m script file was used, and the output of Brockett.m, contained in

Appendix E: , was verified using reference [1]. A summary of the output at sample chord position for the MATLAB version of Brockett's program, and the original data published by Brockett is shown in Table 1. As seen in Table 1, the data agrees within approximately 7 significant figures, which is the number of significant figures expected for the single-precision data type used by FORTRAN. The default for MATLAB is to use double-precision floating-point numbers, which would explain the slight differences between MATLAB calculations, and Brockett's published data.

	Press				
Chord Positition	Brockett	MATLAB Version	% Difference		
(x/C)	Ref [1]	of Brockett			
1	1.000000	1.000000	0.00000%		
0	-0.495333	-0.495331	0.00040%		
Upper Surface					
0.75	-0.099686	-0.099685	0.00100%		
0.5	-0.329138	-0.329138	0.00000%		
0.25	-0.616285	-0.616284	0.00016%		
0.178606	-0.689939	-0.689939	0.00000%		
0.030154	-1.226171	-1.226170	0.00008%		
Lower Surface					
0.030154	0.540715	0.540716	-0.00018%		
0.178606	0.033803	0.033804	-0.00296%		
0.25	-0.037634	-0.037633	0.00266%		
0.5	-0.038729	-0.038728	0.00258%		
0.75	0.042112	0.042113	-0.00237%		
Lift Coeff	0.430129	0.430129	0.00000%		

Table 1: Comparison of Brockett's published data to MATLAB Version of Calculations

Output from the original and the MATLAB version of Brockett's work include multiple tables consisting of Profile Constants and Pressure Distribution information. In addition to the screen output of the data, the MATLAB version saves the data as text files labeled Pressure.txt and Profile.txt, which are saved in the "Data" folder. This data is later read into variables by MATLAB, or may be opened by the user.

3.4 Brockett Analysis Results

To compare the pressure distribution calculated by the Brockett method to the exact solution shown in Figure 3, KT_IN.m was written to read in the ordinates of the normalized Karman-Trefftz foil generated in Figure 2. The X-Y ordinates are cosine-spaced, and the number of points is specified by the 'out_pts' variable in ConfrmlTrans.m. If out_pts is set to 37, this matches the required input location format of Brockett, but is not required. Otherwise, BROCKETT.m will accept format as arbitrary location input. Results obtained from the use of a Karman-Trefftz foil shape is shown in Figure 4.

Figure 4 through Figure 7 shows that Brockett's method predicts the general shape of the pressure distribution and gives an estimate of the minimum pressure for the foil, but does not accurately predict the magnitude. In Figure 4, the actual minimum pressure coefficient, $-Cp_{min}$, is 1.192 at X/C = 0.32. Brockett's analysis predicts that $-Cp_{min} = 1.079$ at X/C = 0.33, which falls short by 9.5%. This error increases as angle of attack increases. Inviscid minimum pressure coefficients predicted by Brockett are compared to analytic results for the Karman-Trefftz foil for various angles of attack in Table 2.

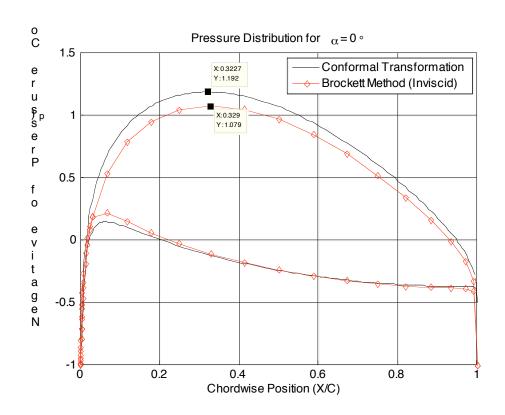


Figure 4: Comparison of Brockett Method to Exact Solution, $\alpha = 0^{\circ}$

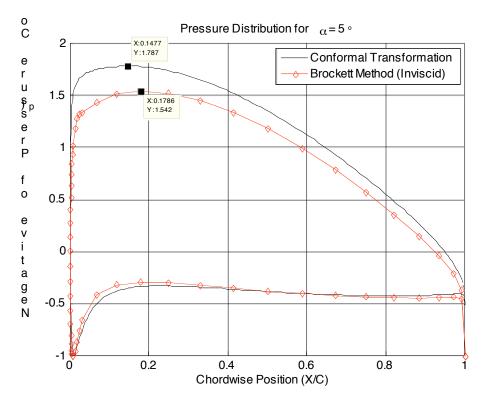


Figure 5: Comparison of Brockett Method to Exact Solution, $\alpha = 5^{\circ}$

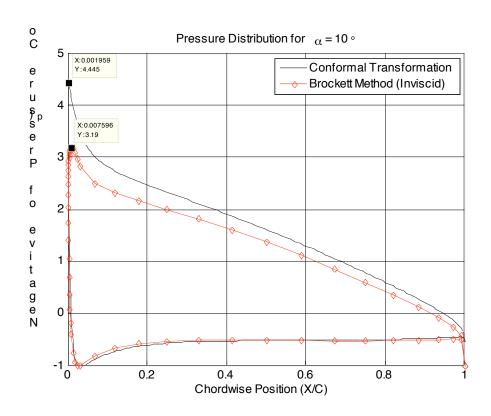


Figure 6: Comparison of Brockett Method to Exact Solution, $\alpha = 10^{\circ}$

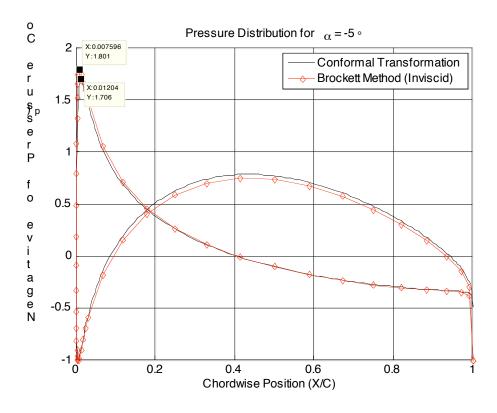


Figure 7: Comparison of Brockett Method to Exact Solution, $\alpha = -5^{\circ}$

	-Cp_m		
_	Brockett	Analytic	Error
-15	11.0498	10.7114	3.16%
-10	5.5078	5.2178	5.56%
-5	1.8012	1.7059	5.59%
-4	1.3052	1.2482	4.57%
-2	0.932	1.017	-8.36%
0	1.0788	1.1919	-9.49%
2	1.2366	1.3938	-11.28%
4	1.4281	1.6374	-12.78%
6	1.6749	2.0243	-17.26%
8	2.32	3.0368	-23.60%
10	3.1896	4.4451	-28.24%
15	6.8295	9.4474	-27.71%

Table 2: Comparison of Brockett's Method of Calculation of Minimum Pressure Coefficient to Exact Solution for a Karman-Trefftz Foil

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4 Introduction to XFOIL

4.1 XFOIL Functionality

XFOIL 1.0 was written by Mark Drela in 1986. The main goal was to combine the speed and accuracy of high-order panel methods with the new fully-coupled viscous/inviscid interaction method used in the ISES code developed by Drela and Giles. A fully interactive interface was employed to make it much easier to use than the traditional batch-type CFD codes. Several inverse modes and a geometry manipulator were also incorporated early in XFOIL's development, making it a fairly general airfoil development system [6].

XFOIL is an analysis and design system for Low Reynolds Number Airfoils. XFOIL uses an inviscid linear-vorticity panel method with a Karman-Tsien compressibility correction for direct and mixed-inverse modes. Source distributions are superimposed on the airfoil and wake permitting modeling of viscous layer influence on the potential flow. Both laminar and turbulent layers are treated with an e⁹-type amplification formulation determining the transition point. The boundary layer and transition equations are solved simultaneously with the inviscid flow field by a global Newton method [7].

4.2 XFOIL Formulation Summary

Details of XFOIL's formulation are presented in reference [7], and will only be summarized here. XFOIL uses a general inviscid airfoil flow field, constructed by the superposition of a free stream flow, a vortex sheet of strength γ on the airfoil surface, and source sheet strength, σ , on the airfoil surface and wake. The airfoils contour and wake trajectory is discretized into flat panels, with panel nodes on the airfoil and wake. Each airfoil panel has a linear vorticity distribution defined by the node value. Each airfoil and wake panel has a constant source strength, which is later related to viscous layer quantities. Requiring the streamfunction to be equal to a constant value at each of the nodes on the airfoil surface results in a system of linear equations that could be solved in combination with the Kutta condition.

XFOIL's viscous formulation was not used extensively in this research, and will not be discussed in detail.

4.3 Adaptation of XFOIL

XFOIL has been in use for many years, and has become a highly regarded analysis tool. This project adapts the improved functionally of XFOIL for use in propeller design by using the algorithms contained within XFOIL to conduct the analysis of a given foil. In particular, XFOIL is used to develop minimum pressure envelopes, or cavitation buckets, as presented by Brockett [4].

XFOIL in its current release (XFOIL 6.94) is a menu driven program, which requires interactive user input and manipulation. In addition, XFOIL generates various output plots to allow graphical display and interface by the user. In order to adapt XFOIL's functionality, XFOIL was converted into a "black-box" calculation tool. XFOIL's menu driven functionality was removed by altering the source code such that all desired operational instructions would be input as command line arguments, rather than menu driven items and direct user input. All of XFOIL's plot and screen output utilities were also disabled. By disabling the plot functionality, calculation speed was improved. Also, removal of XFOIL's plot functionality, allowed simpler compilation of the source code, since no graphical interface was required for the operating system. Finally, XFOIL was altered such that any desired output was written to and saved as a data file, which could be read by MATLAB or opened directly by a text editing program.

4.3.1 Executing XFOIL

XFOIL was altered such that it reads in command line arguments that provide instructions, rather than direct input from the operator via menu options. This allows rapid execution by an external program, such as MATLAB. From the DOS prompt, or by executing a system command, XFOIL can be instructed how to process input data, and which results to save. A simple example of how the modified XFOIL program may be executed as follows. At the DOS command prompt, in a directory containing the xfoil.exe program, the user may type:

```
"xfoil NACA 4415 OPER ALFA 5 OPER CPWR output"
```

The above command instructs XFOIL to use internal definition for a NACA 4415 foil, at an angle of attack of 5°, and write the pressure coefficient (Cp) distribution to a file named "output". When executed, the command line above results in a DOS output of the following:

```
START of XFOIL
START of Menu Loop. Command is: NACA
Using NACA 4415
Max thickness = 0.150043 at x =
                                     0.301
Max camber =
                  0.039999 at x = 0.398
START of Menu Loop. Command is: OPER
OPER loop command: A
Angle of Attack:
                     5.000
Calculating unit vorticity distributions ...
OPER loop complete.
START of Menu Loop. Command is: OPER
OPER loop command: CPWR
OPER loop complete.
```

The DOS output above represents informational items intentionally left in the XFOIL program to allow the user to verify that commands were executed properly. The result of the XFOIL calculations are written to a user specified file. Sample format is as follows. The first column is the X-location, starting at the trailing edge, continues along the upper surface around the nose back to the trailing edge. The second column is the calculated pressure coefficients at the corresponding locations.

```
# x Cp

1.00000 0.48832

0.99329 0.28542

0.98206 0.19383

0.96938 0.11911

... ...

0.95217 0.26832

0.96743 0.28589

0.98105 0.31188

0.99296 0.35546

1.00000 0.48832
```

Details of the structure and format of commands are included in Appendix F:

Modified XFOIL User Guide. In addition, Appendix G: Instruction for compiling modified XFOIL Code, contains additional instructions for obtaining and compiling the source code for the modified version of XFOIL.

4.4 Comparison of XFOIL Calculated Pressure Distributions

In section 3.5, the exact solution to the Karman-Trefftz foil pressure distribution was compared to the method presented by Brockett. Figure 8 to Figure 11 compare the XFOIL calculated pressure distribution, the Karman-Trefftz solution and the Brockett solution. In each instance, it can be seen that the XFOIL solution is nearly identical to the analytic solution. The XFOIL calculations for Figure 8 through Figure 11 were performed by instructing XFOIL to repanel the foil using 50 panels. Although higher panel resolution could be specified (160 is default), 50 was specified to prevent an excessive number of data points.

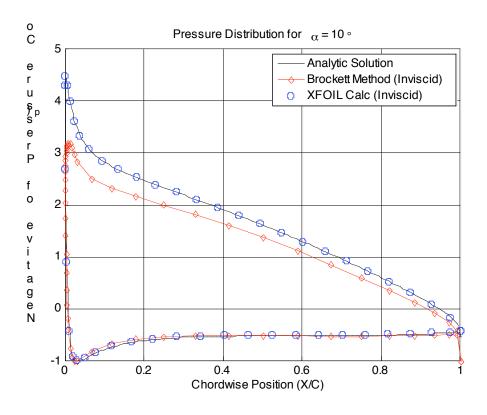


Figure 8: Comparison of XFOIL and Brockett Method to Exact Solution, $\alpha = 10^{\circ}$

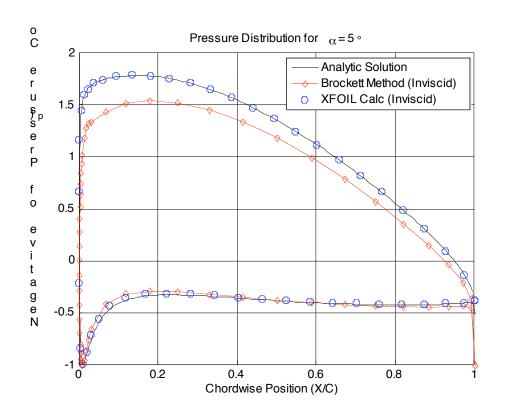


Figure 9: Comparison of XFOIL and Brockett Method to Exact Solution, $\alpha = 5^{\circ}$

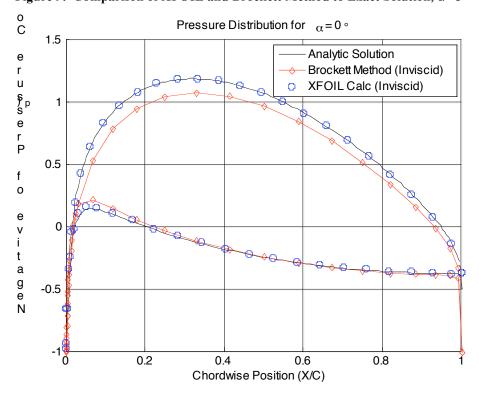


Figure 10: Comparison of XFOIL and Brockett Method to Exact Solution, α =0°

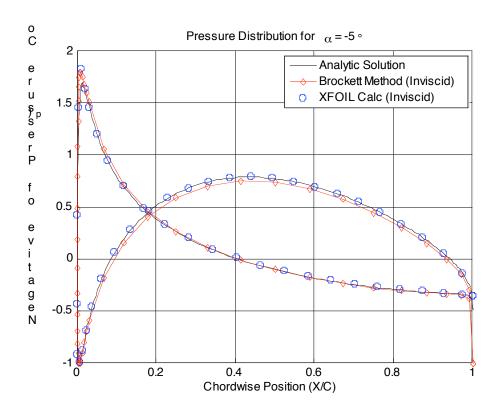


Figure 11: Comparison of XFOIL and Brockett Method to Exact Solution, α =-5°

5 Minimum Pressure Envelope Analysis

5.1 Background

In reference [4], Brockett published minimum pressure envelops for modified NACA-66 sections with NACA A=0.8 camber and BUSHIPS Type I and Type II sections using the calculation method described in section 3. These minimum pressure envelopes were computed for steady two-dimensional flow, with an empirical correction for viscosity. In addition, design charts for selecting "optimum" foils were included.

The work presented here includes a similar analysis method, with calculations performed by XFOIL, allowing the generation of minimum pressure envelopes for an arbitrary foil shape. This was accomplished using MATLAB integrated with the modified version of XFOIL described in section 4.3. Based on the improved accuracy of XFOIL over the method proposed by Brockett as shown in section 4.4, it is ascertained that this method provides a more accurate calculation of the pressure distribution, and location and magnitude of the minimum pressure for the inviscid solution

5.2 Description of Minimum Pressure Envelope Generation

Appendix H: <u>MATLAB Files for Calculation of Minimum Pressure Envelopes</u> contains the MATLAB files that were used to generate the minimum pressure envelopes using the modified XFOIL executable. This script performs various functions described in the following sections.

5.2.1 Foil Shape Generation

Foils may be defined in either of two methods. XFOIL contains built in functions defining NACA 4 and 5-digit series foils. If the user desires to use these NACA foils, then the <code>foil_type</code> variable should be set to "NACA". If NACA series foil shape is desired, the user must also set the variable <code>foil_name</code> to either "FOUR" or "FIVE", depending on which NACA series is desired, and the chordwise position of maximum camber must be specified by the <code>fo_loc</code> variable.

If profile data will be read in from a data file, then the <code>foil_type</code> variable should be set to "LOAD". Foil shape is defined by meanline and thickness information. The data files containing the meanline and thickness information are specified by the user using the <code>mean_type</code> and <code>thick_type</code> variables, which are set to the name of the files containing the meanline and thickness offset values. The data files need to be located in the corresponding "Meanline" and "Thickness" folders. Sample format for these files is contained and described in Appendix I: Meanline and Camber Data File Format.

The makefoil.m function inputs the meanline and camber data from the files specified, and combines the meanline and thickness distributions in the standard method as described by Abbott and Von Doenhoff [8]. In addition, the camber and thickness distributions are scaled if required. The makefoil.m function utilizes MATLAB's spline function to interpolate the required points to define the foil surface. XFOIL requires that foil geometry is specified by defining the X-Y locations along the foil surface from trailing edge, along the upper surface, around the leading edge, and back to the trailing edge along the lower surface. This is accomplished within the makefoil.m function.

The user may specify the number of desired output points to export to XFOIL by specifying the N_parab_eval and N_surf_pts in the makefoil.m script. Care should be used to specify a reasonable number of points, especially along the leading edge. Too many points may cause errors in XFOIL due to excessively small spacing. However, a sufficient number of points to adequately define the foils should be used, provided they are adequately spaced, with more points in regions of higher curvature. N_parab_def specifies the number of points used for creating the spline that defines the nose radius. If less than approximately 20 points are specified, the spline utility fails to produce a smooth output curve. Input and output may be plotted to verify proper definition of surface locations by setting the make_plot variable to 'yes', otherwise it should be set to 'no' to prevent excess plot generation.

5.2.2 User Specifications and Output from XBucket.m

The output from the MATLAB script, XBucket.m, may be specified by the user. The purpose of the script is to produce minimum pressure envelopes for the foil geometry specified by the user. Output plots are a similar format to that of Brockett [4]. Sample output plots show in Figure 12 and Figure 13 below.

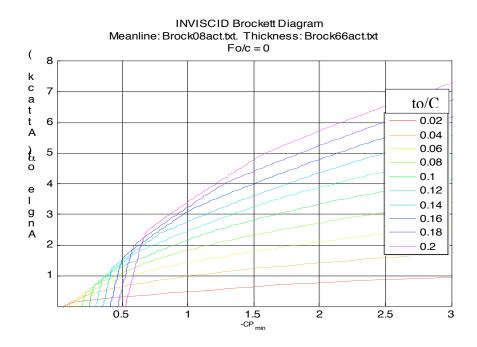


Figure 12: Minimum Pressure Envelopes for NACA 66 Section (TMB Mod. Nose and Tail) with Zero Camber at Various Thicknesses

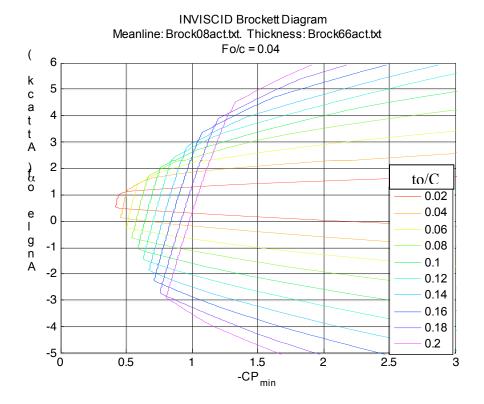


Figure 13: Minimum Pressure Envelopes for NACA 66 Section (TMB Modified) with the NACA a=0.8 Camberline, Having a Maximum Camber Ratio of 0.04 at Various Thicknesses.

XBucket.m generates output as in Figure 12 and Figure 13 based on user specified ranges. The upper and lower bounds of the angle of attack for which calculations and plotting are performed is specified by Alpha lim.

Alpha_delta specifies the resolution, or increment in angle of attack, for which each minimum pressure coefficient is determined. Larger values of Alpha_delta save calculation time, but produce less accurate plots.

Each plot produced is for a specified camber ratio. The desired range and camber ratio increment are specified by the foc_rng and foc_step variable. A separate plot will be produced for each camber ratio from the lower foc_rng value to the upper foc_rng value, in increments of foc_step.

Similarly, on each plot are minimum pressure envelopes for each thickness ratio. The range of values for thickness is specified by toc_rng, in increments of toc step. A separate curve is plotted for each thickness value.

Also, although not a specific concentration of this project, the user may specify that XFOIL's viscous calculation mode be used. In order to conduct viscous calculations, the user must set <code>visc_tog</code> to 1, and specify the desired Reynolds number for calculation. This function has been incorporated for further research. Initial results are not reliable, as XFOIL does not converge consistently. The effect of convergence failure is shown for a typical case in Figure 14. The jagged curve is a result of XFOIL's viscous caluculations failuring to converge when calculating the minimum pressure coefficient for a given angle of attack.

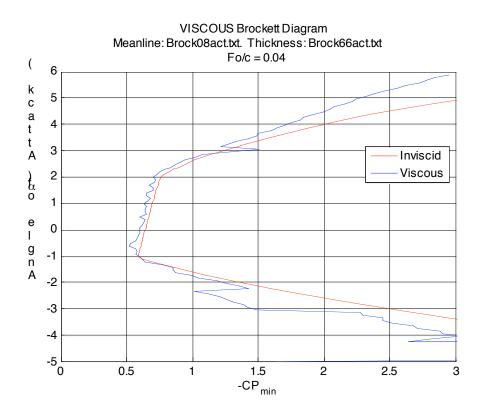


Figure 14: Sample Viscous and Inviscid Minimum Pressure Envelopes Calculated by XFOIL (Reynolds Number = $1*10^6$, 100 Maximum Iterations)

It is believed that small panel size (or excessive number of panels), angle of attack, Reynolds Number, maximum number of iterations and viscous solution acceleration parameter (VACC) are all factors that affect XFOIL convergence. Various combinations were tried to improve comvergence. Better results were obtained when adding the command to repanel the foil with 70 panesl, vice the previous value of 140. Results are shown in Figure 15. Additional research should be

conducted to evaluate the viscous calculation cabability of XFOIL, and determine how to most effectively set parameters that result in smooth, consistent, convergent results.

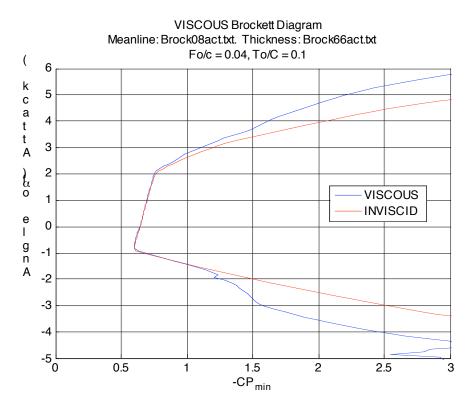


Figure 15: Sample Viscous and Inviscid Minimum Pressure Envelopes

Calculated by XFOIL (Reynolds Number = $1*10^6$, 100 Maximum Iterations, 70 Panels)

5.3 Comparison of Brockett's Method to XFOIL Results

This intent of this research was to create a MATLAB based utility that would reproduce the minimum pressure diagrams published by Brockett [4], which could be later integrated into OpenProp for propeller design. Initial attempts using a simple two-dimensional panel method did not closely match Brockett's published results. As a result, XFOIL was implemented in order to conduct the pressure distribution calculations. XFOIL was chosen due to its highly regarded reputation as an accurate tool for conducting foil analysis and design. Results using XFOIL were still not able to reproduce the data as expected. Finally, the program as published by Brockett in reference [1] was reprogrammed in MATLAB in order to conduct

further comparison. It was this comparison that revealed the noticeable differences between Brockett's results and exact theory for potential flow, as previously shown in Figure 4 through Figure 7.

Figure 16 below illustrates the differences between Brockett's published minimum pressure envelopes, and the XFOIL calculated results. Figure 16 presents minimum pressure envelopes for the NACA 66 (TMB Modified), a = 0.8 meanline. Each individual curve was developed for a camber ratio of 0.06, and a thickness ratio of 0.12.

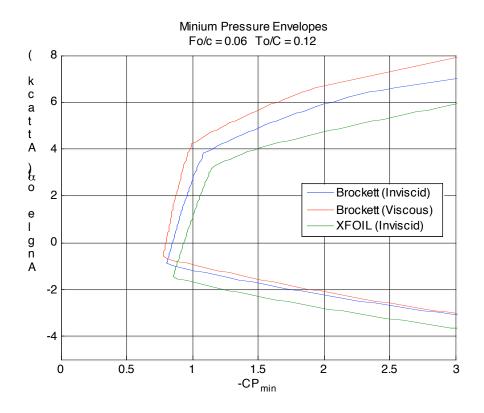


Figure 16: Minimum Pressure Envelope Comparison

Figure 16 illustrates the difference between Brockett (Inviscid) and the XFOIL (Inviscid) solutions. The difference is a result of inaccuracies of the Brockett method to predict the minimum pressure coefficient, as previously discussed in section 4.4, and summarized in Table 2. Specifically, the Brockett method underestimates the magnitude of the minimum pressure coefficient for intermediate and positive angles of attack, which corresponds to the near vertical and upper

portions of the minimum pressure envelope, and overestimates the magnitude of minimum pressure at negative angles of attack. The near vertical portion of the graph represents the region of operation when the minimum pressure occurs in the vicinity of the mid-chord. The upper and lower portions of the envelope correspond to nose cavitation, when the minimum pressure occurs near the leading edge of the foil due to elevated angles of attack.

The trend shown in Figure 16 is typical for all thicknesses and camber ratios. As a result, the overall minimum pressure envelopes as shown in Figure 12 and Figure 13 do not exactly match the published results of Brockett [4]. In addition, Brockett's published minimum pressure envelops for modified NACA-66 sections with NACA a=0.8 camber include an empirical correction for viscosity. The difference between Brockett's potential theory calculation and empirical correction for viscosity is also shown in Figure 16. The magnitude of the difference between Brockett's viscous and inviscid calculations is approximately equal to the magnitude difference between Brockett's inviscid calculation and XFOIL. As a result, it is believed that further investigation should be conducted to account for the viscous effects, and how viscous effect could be accounted for using XFOIL. As previously noted, XFOIL is capable of performing viscous calculations, and that ability was retained in the modified version XFOIL used for this work. However, accurate results were not reliably obtained, and were not evaluated. Further research in this area is recommended, which would greatly enhance the capabilities generated as for this project.

6 OpenProp Implementation Approach

Open-source Propeller Design and Analysis Program (OpenProp) is an open source MATLAB®-based suite of propeller numerical design tools. This program is an enhanced version of the MIT Propeller Vortex Lattice Lifting Line Program (PVL) developed by Professor Justin Kerwin at MIT in 2001. OpenProp v1.0, originally titled MPVL, was written in 2007 by Hsin-Lung Chung and is described in detail in [9]. Two of its main improvements versus PVL are its intuitive graphical user interfaces (GUIs) and greatly improved data visualization which includes graphic output and three-dimensional renderings.

OpenProp was designed to perform two primary tasks: parametric analysis and single propeller design. Both tasks begin with a desired operating condition defined primarily by the required thrust, ship speed, and inflow profile. The parametric analysis produces efficiency diagrams for all possible combinations of number of blades, propeller speed, and propeller diameter for ranges and increments entered by the user. Efficiency diagrams are then used to determine the optimum propeller parameters for the desired operating conditions given any constraints (e.g. propeller speed or diameter) specified by the user.

OpenProp was developed to serve as an open source code for propeller design. While it is currently a tool used in the initial design phase, it is a base program that can be continually expanded to perform detailed design and analysis of sophisticated marine propulsors and turbines. Development of a method of cavitation analysis that could be integrated into OpenProp was a primary motivation for this thesis.

The use of MATLAB provides for integration into the propeller design suite, OpenProp. Integration of cavitation analysis into OpenProp would provide the designer information about cavitation conditions while early in the design process, allowing adjustments to blade geometry to correct deficiencies. Following the design recommendations of Brockett [4], design charts or internal data feedback could provide adjustments to blade geometry. Cavitation prediction could either be

conducted for existing foils or foil geometry could be selected in order to avoid cavitation conditions for a given set of operating conditions.

6.1 Analysis of Existing Foils

For existing propeller, where blade geometry is known, the code presented here could be used to conduct cavitation analysis for the foil. The geometry for the foil can be formatted as required, and may be used as input. To predict cavitation on existing foils, the minimum pressure curve for the propeller geometry at the radial position under investigation should first be generated. Then, based on the given operating conditions (angle of attack and local cavitation number, $\sigma = [p_{\infty} - p_{vapor}]/[1/2pU^2]$), the operating point may be compared to the calculated minimum pressure envelope. By setting the cavitation number equal to the negative of the minimum pressure coefficient, the operating point may be determined. If the operating point falls within the region bound by the minimum pressure envelope, cavitation is assumed not to occur. Cavitation is assumed to occur in the region outside of the minimum pressure envelope.

To analyze a complete propeller blade, it is recommended that a routine be created that analyzes the propeller blade at various radial positions from the hub to the tip at user specified intervals. At each radial position, the geometry must be determined as input. In addition to the minimum pressure coefficient, the pressure distribution along the chord may be calculated and compared to the cavitation number. By determining where the negative of the pressure coefficient is greater than the cavitation number, regions along the propeller where cavitation is predicted could be predicted. These regions could then be used to produce a color coded plot of the surface of the propeller blade, indicating regions were cavitation is predicted to occur.

Margin to cavitation could also be determined. For propellers that are predicted not to cavitate, the operating angle of attack can be compared to the angles of attack at the upper and lower bounds of the minimum pressure envelope for the cavitation number. The difference between the operating angle of attack and the angles of

attack at the envelope boundaries gives an indication of how close the propeller is to cavitation based on expected operating conditions. This information can also be used to predict how far from design conditions the propeller may be operated before the onset of cavitation. Varying inflow would be an example of off design conditions that could be analyzed using the margin to cavitations. If the inflow is known to vary by 2° around the circumference, then as long as the margin to cavitation is greater than 2°, cavitation would not be expected to occur due to varying inflow.

6.2 Geometric Design to Prevent Cavitation

Rather than analyzing an existing propeller, minimum pressure envelopes could be used as an aid to the designer in producing a propeller blade geometry that is optimized to prevent cavitation. 'Optimum' foil geometry, as described by Brockett, allows the greatest total angle change without occurrence of cavitation for a given cavitation number. For symmetric foils (Figure 12), the optimum foil is the one for which the minimum pressure envelope is the widest at the given –Cp_{min}. In other words, it is the thickness which provides the greatest envelope width for the given operating conditions.

For cambered foils, there are two separate curves that bound the minimum pressure envelopes, one for the upper portion, and one for the lower. These bounding curves are shown below in Figure 17. The solid line shows the bounds for the upper portion and the dashed line shows the bound for the lower. These charts can be use to aid in selection of the appropriate thickness and camber based on operating conditions.

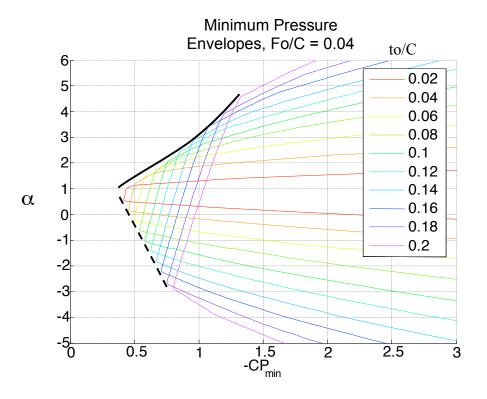


Figure 17: Minimum Pressure Envelopes for Design

Based on design specifications (lift coefficient and cavitation number), the designer has the option of two methods for determining an angle of attack for which to place the foil. The angle of attack may be specified as the ideal angle of attack, or the angle that maximizes the margin to cavitation, located halfway between the minimum pressure envelope curve for the given thickness at the specified cavitation number.

From thin airfoil theory, the idea angle of attack, α_i , is defined as the angle of attack for which the coefficient $A_o = 0$. A_o is the angle of attack dependent coefficient in the Fourier series expansion of df/dx (camberline slope) [5]. Equations (6.1) and (6.2) below provide details.

$$\alpha_i = \frac{1}{\pi} \int_0^{\pi} \frac{df}{dx} dx^{\tilde{x}}$$
 (6.1)

$$x = -\frac{c}{2}\cos(x) \tag{6.2}$$

The ideal angle of attack is tabulated for many types of foils, or may be calculated using the relations above. Once determined, the ideal lift coefficient, which corresponds to the lift coefficient at the ideal angle of attack, should be evaluated, and may be obtained from XFOIL. Once the ideal lift coefficient is determined, the camber ratio may be determined by scaling the tabulated camber value by the same ratio as the desired to ideal lift coefficients.

$$\frac{\binom{fo/C}{C}}{\binom{fo/C}{l_{able}}} = \frac{Cl_{desired}}{Cl_{ideal}}$$
(6.3)

Using the minimum pressure envelopes for the calculated camber, the thickness ratio can be determined based on the upper bounding curve of Figure 17 by entering the graph at the operating cavitation number. Numerically, this could be accomplished by one of two methods. If the location of the "knuckles" of the envelopes can be identified, the bounding curve for the minimum pressure envelopes can be determined, similar to the bounds in Figure 17. Defining the bounding surface as a function of thickness ratio would allow the designer to directly calculate the thickness that results in the widest envelope for the given cavitation number. This method was attempted, but determining the location of the "knuckles" was difficult and a reliable method was not found. Rather than determining a function that describes the bounds of the minimum pressure envelopes, it is recommended that the margin to cavitation be calculated incrementally for each of the thicknesses, until a maximum is found for the specified cavitation number. Finally, the width of the envelope can be determined by finding the intersection of the cavitation number with the lower portion of the minimum pressure envelope for the given thickness. The envelope width can then be compared to any radial inflow variation to evaluate propeller performance with varying inflow.

Rather than assuming the foil be operated at the ideal angle of attack, a method that maximizes cavitation margin could be used. For this purpose, cavitation margin is defined as the magnitude of the difference between the operating angle of attack, and the angle at which cavitation is predicted. In order to maximize the cavitation margin, the operating angle of attack should be exactly in the middle of the upper and lower legs of the minimum pressure envelope.

An approach to determining angle of attack, camber and thickness, would be to start by specify foil type, cavitation number, and required lift coefficient. Using the foil type, minimum pressure envelopes for varying camber and thickness should be produced. Start with the envelopes of minimum camber and the negative of the minimum pressure coefficient equal to the cavitation number. The thickness producing the maximum envelope width would then be determined. The, the values for the angles of attack corresponding to the maximum with at the specified –Cp should be determined. The operating airfoil angle of attack should be the midpoint between these values.

Once an initial camber, thickness, and angle of attack have been specified, the resultant lift coefficient should be calculated and compared to desired lift coefficient as specified in the design. If the calculated lift coefficient is less than required, the camber ratio should be increased incrementally and the thickness, angle of attack, and lift coefficient recalculated until desired lift coefficient is achieved.

7 Conclusion

A method of generating minimum pressure envelopes using XFOIL was created. By modifying the source code, a version of XFOIL that does not require user interaction was created. Using MATLAB to interface with XFOIL, minimum pressure envelopes for an arbitrary foil shape can be generated, provided offset data is available for foil geometry.

The minimum pressure envelopes created as a result of the XFOIL calculations were compared to published work by Brockett. It was found that the two-dimensional panel method of XFOIL could more accurately estimate the potential flow solution for a Karman-Trefftz foil than the approximate conformal transformation method used by Brockett. Although XFOIL includes the ability to conduct calculations for a viscous fluid, additional work is required in order to evaluate the limits for which XFOIL's viscous mode will reliably converge.

The code developed as part of this thesis is intended to be used for further integration into OpenProp. Integration into OpenProp will allow the user to both conduct cavitation analysis and prediction for existing foils, as well as allow the designer to consider cavitation in the design process, and select foil geometry that will prevent cavitation.

7.1 Recommendations for Future Work

7.1.1 Viscous Calculations

In order to utilize the benefits of the highly accurate potential flow solution available from XFOIL, the effects of viscosity must be reevaluated, and accounted for. The empirical modification used by Brockett depends upon specifying an experimental lift coefficient for each angle of incidence, and can be determined from a lift-curve slope and angle of zero lift using the following equation:

$$C_L = 2\pi\eta \left(\alpha - \alpha_{0_e}\right) \tag{7.1}$$

Where η is the lift-curve slope coefficient and α_{0e} is the experimental angle of zero lift. It has been experimentally shown [8] that η and α_{0e} are independent for high Reynolds numbers (>6x10⁶). This method should be reevaluated and compared to the manner by which viscous calculations are conducted in XFOIL. XFOIL's viscous mode should be integrated into the methods presented here for calculating minimum pressure envelopes, if found to be accurate. In addition, the parameters affecting convergence of XFOIL's viscous calculation should be evaluated to ensure proper problem formulation and evaluation.

7.1.2 OpenProp Integration

The design approaches outlined in section 6 should be developed and integrated into OpenProp. Once developed, these methods would provide great benefit and enhance the utility of the OpenProp design suite. Program code that performs the basic functions required to integrate cavitation design were developed in this thesis. In particular, the development of the modified version of XFOIL allows rapid calculation of complex foil geometries, and simplifies the method of retrieving data from XFOIL and entering that data into MATLAB. In addition, the functions generated in project provide a great starting point from which to develop the functionality and usefulness of OpenProp.

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Appendix A: Matlab Script for Conformal Transformation of Karman-Trefftz Foil.

```
% Code Developed by Chris Peterson to calculate and display Conformal
  Transformation of a Karman-Trefftz foil. Intended to be used to
   compare values for different methods of calculating surface velocites
    for airfoils.
clc; clear all; close all;
%User defined Data
           = 1;
                             %Free Stream Velocity
alpha_deg = 0; %Angle of Attack (Degrees)
xc = -0.10; %Circle Center Location (<0)
yc = 0.150; %Circle Center (>0 adds + ca
                             %Circle Center (>0 adds + camber)
           = 10;
                             %Tail Angle
tau
alpha = deg2rad(alpha deg);
%Display parameters
                        %Number of Streamlines to plot
%Z-plane X-Y Range
No_strm = 21;
range = 3; %Z-plane X-Y Range
strm_strt = -3; %X-Location for streamline start
div = 0.1; %Grid spacing for velocity vectors on Z-plane
%Calculation of properties
                                               %Angle to rear stagnation point
beta = atan(-yc/(1-xc));
beta_deg = rad2deg(beta);
rc = sqrt((a-xc)^2 + yc^2);
Gamma_calc = 4*pi*rc*U*sin(beta-alpha);
%Beta in degrees
%Calculate radius of circle
%Kutta condition requirement
Gamma = Gamma calc;
                                                %Set circulation to required
lam = 2-tau/180;
                                              %Trailing egge to lamba calculation
%Generate Z-plane Plot with streamlines and velocity vectors
[X,Y] = meshgrid(-range:div:range,-range:div:range); %Create location mesh
r = sqrt((X-xc).^2 + (Y-yc).^2); %Radius at mesh locations
%Calculate angle theta to mesh locations, 0 <= theta < 2*pi</pre>
for j=1:length(X)
    for k=1:length(X)
         if X(j,k) >= xc
             theta(j,k) = atan((Y(j,k)-yc)/(X(j,k)-xc));
         elseif X(j,k) < xc
             theta(j,k) = atan((Y(j,k)-yc)/(X(j,k)-xc)) + pi;
         end
    end
%Calculate velocity components u, v based on potential theory
u = U*cos(alpha) - (U.*((rc./r).^2).*cos(2.*theta - alpha))...
    - Gamma.*sin(theta)./(2.*pi.*r);
v = U*sin(alpha) - (U.*((rc./r).^2).*sin(2.*theta - alpha))...
    + Gamma.*cos(theta)./(2.*pi.*r);
%Calculate location of stagnation points
theta s1 = asin(Gamma/(4*pi*rc*U)) + alpha;
theta_s2 = asin(-Gamma/(4*pi*rc*U)) + alpha - pi;
x_st1 = rc*cos(theta_s1)+xc;
y_st1 = rc*sin(theta_s1)+yc;
x_st2 = rc*cos(theta_s2)+xc.
x_st2
           = rc*cos(theta s2)+xc;
y st2 = rc*sin(theta s2)+yc;
%Define point on circle
```

```
x circ = xc + rc*cos(0:pi/21:2*pi);
y circ = yc + rc*sin(0:pi/21:2*pi);
z circ = x circ + i*y circ; %z is complex coordinates of circle
%Eliminates points inside circle for vector plot (large values near
singularities)
u \mod = u;
v \mod = v;
for j = 1:length(X)
    for k = 1:length(X)
        if (X(j,k)-xc)^2 + (Y(j,k)-yc)^2 < rc^2
           u \mod(j,k) = 0;
            v \mod (j,k) = 0;
        end
    end
end
    Plot Z-plane, with circle, stagnation points, velocity vectors and
   streamlines.
figure()
orient landscape;
axis equal; hold on; grid on; ylim([-range range]); xlim([-range range]);
title({'Z-plane';'Potential Flow Around a Circle'});
set(gca,'YTick',-range:range);set(gca,'XTick',-range:range);
streamline(stream2(X,Y,u,v,strm strt*ones(No strm,1)...
    ,-range:2*range/(No strm-1):range)); %Plots streamlines
plot(x circ, y circ, 'k')
                                           %Plots circle
plot(x circ, y circ, 'k.')
                                           %Plots circle points
plot(xc, yc, 'r+')
                                           %Plots circle center
plot(x_st1, y st1, 'ko')
                                           %Plots stagnation point 1
plot(x_st2, y_st2, 'ko')
                                           %Plots Stagnation point 2
% quiver(X,Y,u_mod,v_mod, 'g');
                                             %Plots Vectors
%Map surface of ccircle to Zeta-plane
Zeta circ = lam*a*((z circ+a).^lam + (z circ-a).^lam)...
    ./((z circ+a).^lam - (z circ-a).^lam);
%Routine to find velocities and -Cp on foil surface
theta = 0:2*pi/n pts:2*pi-pi/n pts; %Defines theta incremented 0->2*pi
      = xc + rc*cos(theta); %X location in Z-plane
= yc + rc*sin(theta); %Y location in Z-plane
X Z
       = yc + rc*sin(theta);
                                   %Y location in Z-plane
y_z
      = U*cos(alpha)... %X velocity in Z-plane
u z
           - (U.*cos(2.*theta - alpha)) - Gamma.*sin(theta)./(2.*pi.*rc);
       = U*sin(alpha)...
                                  %Y velocity in Z-plane
           - (U.*sin(2.*theta - alpha)) + Gamma.*cos(theta)./(2.*pi.*rc);
                                   %Complex velocoity in Z-plane
z z = x z + i.*y z;
%Transform Surface Locations & Velocities to Zeta Plane
       = lam*a.*...
                                   Complex coords Zeta = f(z)
            ((z z+a).^lam + (z z-a).^lam)./((z z+a).^lam - (z z-a).^lam);
                                   %X location in Zeta-plane
x zeta = real(Zeta);
y zeta = imag(Zeta);
                                   %Y location in Zeta-plane
dzeta dz = (4*(lam*a)^2)...
                                   %D(Zeta)/Dz
            *( ((z z-a).^(lam-1)) .* ((z z+a).^(lam-1)))...
            ./(((zz+a).^lam) - ((zz-a).^lam)).^2);
vel zeta = (u z - i.*v z)./dzeta dz; %[u-iv] zeta = [u-iv] x/Dzeta/Dz
u zeta = real(vel zeta);
                                %X velocity in Zeta-plane
v zeta = -imag(vel zeta);
                                  %Y velocity in Zeta-plane
q zeta = sqrt(u zeta.^2 + v zeta.^2); %Zeta-Velocity Magnitude
cp zeta = 1-(q zeta./U).^2; %Zeta pressure coefficient
%Create plot of Zeta plane
```

```
figure(); grid on; hold on; axis equal; orient landscape;
xlim([-range range]);ylim([-range+1 range-1]);
title({'\zeta-Plane';'Potential Flow Around Mapped Foil'});
plot(Zeta circ, 'k')
%plot(Zeta circ, 'k.')
%Find Z coodinates in Z-Plane based on required spacing in Zeta-plane
%in order to calculate U-V components in mesh spacing for streamline plot
[Xgrd zeta, Ygrd zeta] = ...
                                          %Create location mesh
    meshgrid(-range:div:range,-range:div:range);
Zetagrd = Xgrd zeta + i*Ygrd zeta;
Z \text{ grd} = -a.*((((Zetagrd-lam)./(Zetagrd+lam)).^(1/lam))+1)...
    ./((((Zetagrd-lam)./(Zetagrd+lam)).^(1/lam))-1);
X \text{ grd} = \text{real}(Z \text{ grd});
Y = grd = imag(Z grd);
            = sqrt((X grd-xc).^2 + (Y grd-yc).^2); Radius at mesh locations
r grd
%Calculate angle theta to mesh locations, 0 <= theta < 2*pi</pre>
for j=1:length(X grd)
    for k=1:length(X grd)
        if X \operatorname{grd}(j,k) >= xc
             theta grd(j,k) = atan((Y grd(j,k)-yc)/(X grd(j,k)-xc));
        elseif X(j,k) < xc
             theta grd(j,k) = atan((Y grd(j,k)-yc)/(X grd(j,k)-xc)) + pi;
        end
    end
end
%Calculate velocity components u, v based on potential theory
u grd z = U*cos(alpha) - (U.*((rc./r grd).^2).*cos(2.*theta grd - alpha))...
    - Gamma.*sin(theta grd)./(2.*pi.*r grd);
v \text{ grd } z = U*\sin(alpha) - (U.*((rc./r \text{ grd}).^2).*\sin(2.*theta \text{ grd} - alpha))...
    + Gamma.*cos(theta_grd)./(2.*pi.*r_grd);
for j = 1:length(X grd)
    for k = 1:length(X grd)
        if (X \text{ grd}(j,k)-xc)^2 + (Y \text{ grd}(j,k)-yc)^2 < rc^2
            u \operatorname{grd} z(j,k) = 0;
            v \text{ grd } z(j,k) = 0;
        end
    end
end
dzeta_dz_grd = (4*(lam*a)^2)...
                                              %D(Zeta)/Dz
             *( ( (Z grd-a).^(lam-1)) .* ((Z grd+a).^(lam-1)) )...
             ./((((Z grd+a).^lam) - ((Z grd-a).^lam)).^2);
vel grd zeta = (u grd z - i.*v grd z)./dzeta dz grd;
u zeta grd = real(vel grd zeta);
                                              %X velocity in Zeta-plane
v zeta grd = -imag(vel_grd_zeta);
                                              %Y velocity in Zeta-plane
%quiver(Xgrd zeta ,Ygrd zeta, u zeta grd, v zeta grd, 'g')%Plots Vectors
streamline(stream2(Xgrd zeta, Ygrd zeta, u zeta grd, v zeta grd,...
    strm strt*ones(No strm, 1), -range: 2*range/(No strm-1):range)); %Plots
streamlines
%Create and save plot of minimum pressure distribution
figure(); hold on; grid on; xlim([0 1])
xlabel('Chordwise Position (X/C)')
ylabel('Negative of Pressure Coefficient (-C p)')
title(['Pressure Distribution for \alpha = ', num2str(alpha deq),'\circ'])
plot((x zeta-min(x zeta))./(max(x zeta)-min(x zeta)), -cp zeta, 'k.-')
saveas(qcf,'Trefftz.fiq')
close();
%Scales foil to Chord lenght of 1.
```

```
chord = (\max(x zeta) - \min(x zeta));
y zeta scl = y zeta./(max(x zeta)-min(x zeta));
%Locates nose location and index X and \overline{Y}.
[x_nose, i_nose] = min(x zeta scl);
[x tail, i tail] = max(x zeta scl);
%Breaks scaled locations into upper and lower surfaces
x US = [x zeta scl(i tail:end) x zeta scl(1:i nose)];
y US = [y zeta scl(i tail:end) y zeta scl(1:i nose)];
x LS = x zeta scl(i nose:i tail);
y LS = y zeta scl(i nose:i tail);
%Defines X locations, cos-spaced, to be used for output
x spl = (1+cos(0:2*pi/(out pts-1):2*pi))/2;
*Splines Upper and Lower surfaces and evaluates at x spl locations
spl US = spline(x US, y US);
spl LS = spline(x_LS, y_LS);
y spl = [ppval(spl US, x spl(1:ceil(out pts/2))) ...
    ppval(spl LS, x spl(ceil(out pts/2)+1:end))];
%Summary plot to compare input, output, and spline functions
figure();hold on; axis equal;
fnplt(spl US, 'r')
fnplt(spl LS, 'g')
plot(x US, y US, 'k.')
plot(x LS, y LS, 'k.')
plot(x spl, y spl, 'bo')
legend('US Spline', 'LS Spline', 'US Data', 'LS Data', 'Output Points')
%Saves splined output point and angle of attack to file x output
save('x output', 'x spl', 'y spl', 'alpha deg')
%Writes splined output X and Y locations to data file trefxy
fid = fopen('trefxy', 'w');
for i =1:length(x spl)
    fprintf(fid, '%12.8f %12.8f\n', x spl(i), y spl(i));
end
fclose(fid);
%run brockthesis %Starts Brockett's thesis for comparison of data.
```

Appendix B: MATLAB Code of Brockett's Work (Brockett.m)

%ADAPTATION OF BROCKETT'S THESIS WORK. Code Modified by Chris Peterson. %Code allows user specified input, and plots pressure distribution for %given input. clear all; clc; prnt2scrn = 1;%Turn on (1) or off (0) screen output run CfmlInput %Allows user specified setting and data input. %Preallocate memory for Improved Speed CO = zeros(1,18); SO = zeros(1,18); X = zeros(1,18); ANTRP=zeros(1,12);CNT=zeros(1,12);XA=zeros(1,12);SNT=zeros(1,12);COL=zeros(1,17); COT=zeros(1,17); Z1=zeros(12,17); Z2=zeros(12,17); Z3=zeros(12,17);Z4=zeros(12,17); EE=zeros(1,NX); DD=zeros(1,37); if IDEN == 0 SY=zeros(1,19);VL=zeros(1,19);elseif IDEN > 0 SY=zeros(1,36);VL=zeros(1,36); end CALCULATION OF CONSTANTS AN=18.0;for I=1:18 TA=(I-1)*.17453293;CO(I) = cos(TA);SO(I) = sin(TA);X(I) = .5*(1.+CO(I));SO(19) = 0.;CO(19) = -1.;X(19) = 0.;for I=20:37; IA=38-I; X(I) = X(IA);CO(I) = CO(IA);SO(I) = -SO(IA);end INTERMEDIATE POINTS AND CORRESPOINDING X VALUES for I=1:9 ANTRP(I) = (I) \star .017453293; ANTRP (10) = 12.5 * .017453293;ANTRP (11) = 15.0 * .017453293;

```
ANTRP(12)=17.5*.017453293;
for I=1:12
    CNT(I) = cos(ANTRP(I));
    XA(I) = .5*(1-CNT(I));
    SNT(I) = sin(ANTRP(I));
end
   CALCULATION OF VECTORS USED TO OBTAIN SLOPE AND VELOCITY
for I=1:2:17
   COL(I) = -1/(AN*(1-CO(I+1)));
COEF = 1;
for I=1:17
   COEF = -COEF;
    COT(I) = COEF*SO(I+1)/(1-CO(I+1))*0.5;
end
for I=1:12
    COEF
          = 1;
          = cos(18*ANTRP(I));
    SNNT = sin(18*ANTRP(I));
    for J=1:17
       COEF=-COEF;
        TA = (-CNT(I)-CO(J+1));
       TB = (COEF*CNNT-1)/36;
       TC = COEF*SNNT*0.5;
       TD = TA*TA;
       TE = (1+CO(J+1)*CNT(I))/TD;
       TF = SO(J+1)*SNT(I)/TD;
       TD = COEF*CNNT*0.5/TA;
       Z1(I,J) = TB*TF+TC*SO(J+1)/TA;
       Z2(I,J) = TB*TE+TC*SNT(I)/TA;
       Z3(I,J) = (TC/18.)*TF-TD*SO(J+1);
       Z4(I,J) = (TC/18.)*TE-TD*SNT(I);
   end
end
   READ INPUT (REPLACED WITH FUNCTION ARGS)
응C
     ARBITRARY INPUT SUBROUTINE
if IPM > 0
    if prnt2scrn == 1
       fprintf(' INPUT AT ARBITRARY X VALUES\n')
    end
    if IDEN > 0 % (Not symmetric)
        if ILK > 0 %(:INPUT TAU, RHO, RHO)
           if prnt2scrn == 1
               fprintf(' THICKNESS CAMBER NOSE RADIUS\n')
               fprintf('%12.6f',TAO,F,RHO)
               fprintf('\n\n')
                                      YT
                                                    YC DYC/DX\n') %
               fprintf('
PRINT 31
           end
           RHO = RHO*TAO^2;
```

```
for I=1:NX
        AT = AT in(I);
        YT = YT_in(I);
        YC = YC in(I);
        YCP = \overline{YCP} \text{ in (I)};
        IA=2*NX-I;
        if AT > 0
             if YCP ~= 0
                 THT = atan(YCP*F);
SA = sin(THT)*YT*TAO;
CA = car(THT)
                 CA
                         = cos(THT)*YT*TAO;
                 CC(I) = AT-SA;
                 Y(I)
                        = YC*F+CA;
                 CC(IA) = AT + SA;
                 Y(IA) = YC*F-CA;
             elseif YCP == 0
                 Y(I) = YC*F+YT*TAO;

Y(IA) = YC*F-YT*TAO;
                 CC(I) = AT;
                 CC(IA) = AT;
             end
        elseif AT == 0
             THT = atan(YCP*F);
             Y(I) = RHO*sin(THT);
             YN = Y(I);
             CC(I) = -RHO*(1.-cos(THT));
             XN = CC(I);
        end
        if prnt2scrn == 1
             fprintf('%12.6f',AT,YT,YC,YCP)
             fprintf('\n')
        end
    end
    NX = 2*NX-1;
elseif ILK == 0
    if prnt2scrn == 1
        fprintf(' X
                                       Y\n')
        fprintf('%12.6f', XN, YN)
        fprintf('\n')
        for I = 1:NX
             CC(I) = CC in(I);
             Y(I) = Y in(I);
             fprintf('%12.6f',CC(I),Y(I))
             fprintf('\n')
        end
    else
        for I = 1:NX
             CC(I) = CC in(I);
             Y(I) = Y_{in}(I);
        end
    end
end
       = 37;
IMS
       = 1-XN;
       = atan(YN/B);
      = sin(AWK);
SA
CA
       = cos(AWK);
```

```
AWK = AWK*180/pi;
        if prnt2scrn == 1
            fprintf('\n\nROTATED AND SHRUNK INPUT\n')
            fprintf('ANGLE OF ROTATION= %9.6f DEG, \n', AWK)
            fprintf(' NOTE: THIS ANGLE WILL BE ADDED TO EACH OF THE INPUT
ANGLES\n')
            fprintf('
                                                 PHI, DEG\n')
        end
        for I = 1:NX
            CC(I) = (CC(I)-XN)/B;

Y(I) = (Y(I)-YN)/B;
            ALTER = CC(I);
            CC(I) = (CC(I)*CA-Y(I)*SA)*CA;
            Y(I)
                   = (Y(I)*CA+ALTER*SA)*CA;
        end
        ND = NX-1;
        A = (NX+1)/2;
        for I=2:ND
            R
                   = I;
            EE(I) = 2*CC(I)-1;
            if EE(I) ~= 0
                EE(I) = atan(sqrt(abs(1-EE(I)^2))/EE(I));
                if (B-A) <= 0
                    if (CC(I) - .5) < 0
                        EE(I) = EE(I) + pi;
                    end
                elseif (B-A) > 0
                    if (CC(I) - 0.5) < 0
                        EE(I) = pi+abs(EE(I));
                     elseif (CC(I)-0.5) > 0
                         EE(I) = 2*pi - EE(I);
                    elseif (CC(I)-0.5) == 0
                        EE(I) = 1.5*pi;
                    end
                end
            elseif EE(I) == 0
                if (B-A) == 0
                    error('ERRONEOUS INPUT')
                elseif (B-A) < 0;
                    EE(I)=pi/2;
                elseif (B-A) > 0;
                    EE(I) = 1.5*pi;
                end
            end
        end
        EE(1) = 0;
        EE(NX) = 2*pi;
        for I=1:NX
            A=EE(I)*180/pi;
            if prnt2scrn == 1
                fprintf('%12.6f',CC(I),Y(I),A)
                fprintf('\n')
            end
        end
    elseif IDEN == 0
        if prnt2scrn == 1
            fprintf(' X
                                       Y PHI, DEG\n')
```

```
end
    ND=NX-1;
    for I=1:NX
        CC(I) = CC in(I);
        Y(I) = Y in(I);
    end
    EE (1)
          =0;
    if prnt2scrn == 1
        fprintf('
                       Χ
                                             PHI, DEG\n')
        fprintf('%12.6f',CC(1),Y(1),EE(1))
        fprintf('\n')
    end
    for I = 2:ND
        EE(I) = 2*CC(I)-1;
        if EE(I) \sim = 0
            EE(I) = atan(sqrt(abs(1-(EE(I))^2))/EE(I));
            if (CC(I) - 0.5) < 0
                EE(I) = EE(I) + pi;
            elseif (CC(I)-0.5) == 0
                EE(I) = pi/2;
            end
        elseif EE(I) == 0
            EE(I) = pi/2;
        end
           = EE(I)*180./pi;
        if prnt2scrn == 1
            fprintf('%12.6f',CC(I),Y(I),A)
            fprintf('\n')
        end
    end
    EE(NX) = pi;
           = 180;
    if prnt2scrn == 1
        fprintf('%12.6f',CC(NX),Y(NX),A)
        fprintf('\n')
    end
    IMS
           = 19;
    AWK
           = 0;
end
        = 1;
CC(1) = Y(1);
        = 0;
Υ1
        = Y(2) - Y(1);
Y2
        = Y(3) - Y(1);
Υ3
        = Y(4) - Y(1);
        = (Y1*EE(3)-Y2*EE(2))/(EE(2)*EE(3)*(EE(2)-EE(3)));
        = (Y2*EE(4)-Y3*EE(3))/(EE(4)*EE(3)*(EE(3)-EE(4)));
A3
        = (A-B)/(EE(2)-EE(4));
Α2
        = A-A3*(EE(2)+EE(3));
Α1
        = Y1/EE(2)-EE(2)*(A2+A3+EE(2));
        = R + pi/18;
I
        = I + 1;
CC(I) = Y(1) + R* (A1 + R* (A2 + R*A3));
if (R-EE(2)) <= 0
```

R

В

R

```
while (R-EE(2)) \ll 0
        R
             = R + pi/18;
                = I + 1;
        CC(I) = Y(1) + R* (A1 + R* (A2 + R*A3));
    end
end
NP = NX-2;
Y1 = Y(ND) - Y(NX);
Y2 = Y(NP) - Y(NX);
N3 = NX - 3;
Y3 = Y(N3) - Y(NX);
R
   = 0;
X1 = EE(ND) - EE(NX);
X2 = EE(NP) - EE(NX);
X3 = EE(N3) - EE(NX);
Α
   = (Y1*X2-Y2*X1) / (X1*X2*(X1-X2));
В
   = (Y2*X3-Y3*X2) / (X3*X2*(X2-X3));
A3 = (A-B)/(X1-X3);
A2 = A-A3*(X1+X2);
A1 = Y1/X1-X1*(A2+A3*X1);
I = IMS;
R = R-pi/18;
   = I-1;
Ι
CC(I) = Y(NX) + R*(A1 + R*(A2 + R*A3));
  = R+EE(NX);
if (A-EE(ND)) > 0
    while (A-EE(ND)) > 0
        R = R - pi/18;
        I = I-1;
        CC(I) = Y(NX) + R*(A1 + R*(A2 + R*A3));
           = R + EE(NX);
    end
end
for I = 2:IMS
    R = (I-1)*pi/18;
    for J = 2:NP
        if (R-EE(J)) > 0
            if (R-EE(J+1)) < 0
                JP = J-1;
                X1 = EE(J) - EE(JP);
                X2 = EE(J+1) - EE(JP);
                X3 = EE(J+2)-EE(JP);
                Y1 = Y(J) - Y(JP);
                Y2 = Y(J+1)-Y(JP);
                Y3 = Y(J+2) - Y(JP);
                A = (Y1*X2-Y2*X1) / (X1*X2*(X1-X2));
                B = (Y2*X3-Y3*X2) / (X3*X2*(X2-X3));
                A3 = (A-B)/(X1-X3);
                A2 = A-A3*(X1+X2);
                A1 = Y1/X1-X1*(A2+A3*X1);
                    = R-EE(JP);
                CC(I) = Y(JP) + R*(A1 + R*(A2 + R*A3));
            end
        elseif(R-EE(J)) == 0
            CC(I) = Y(J);
        end
    end
```

```
end
    if IDEN == 0
       for I = 1:18
           IA = 38-I;
           CC(IA) = -CC(I);
       end
    end
    CC(IMS) = Y(NX);
    if prnt2scrn == 1
       fprintf('\n\nINPUT AT REQUIRED X VALUES\n')
       fprintf('
                  INDEX
                               X
                                        YU
                                                        YL\n')
    end
    for I = 1:19
       ΙA
             = 38-I;
             = CC(I);
       Y(I)
       Y(IA) = CC(IA);
             = I-1;
       J
       Input at reqd x(I,:) = [J X(I) Y(I) Y(IA)];
       if prnt2scrn == 1
           fprintf('%12.6f',J,X(I),Y(I),Y(IA))
           fprintf('\n');
       end
   end
              = 0;
    Y(19)
    if prnt2scrn == 1
      fprintf(' NOTE: LE AND TE ORDINATES SET=0\n')
   ABA
          = Y(1);
   Y(1) = 0;
elseif IPM == 0
    AWK = 0;
    if IDEN == 0
       Y(19) = 0;
       ABA = Y(1);

Y(1) = 0;
       for I = 1:18
           IA = 38-I;
Y(IA) = -Y(I);
       end
    elseif IDEN > 0
       Y(37) = -Y(1);
             = Y(1);
       ABA
       Y(1) = 0;
    end
end
% CALCULATION OF PROFILE SLOPE AND COTANGENT INTEGRAL
if IDEN < 0
   ERROR('ERRONEOUS INPUT');
elseif IDEN == 0
   MAD = 19;
elseif IDEN > 0
   MAD = 36;
for I=1:MAD
```

```
SY(I) = 0;
    VL(I) = Y(I) *9;
    for J=1:17
        LA = I-J;
        if LA <= 0;
            LA = 36+LA;
        end
        KB=I+J;
        if (KB-36) > 0;
            KB = KB - 36;
        end
        SY(I) = SY(I) + (Y(LA) - Y(KB))*COT(J);
        VL(I) = VL(I) + (Y(LA) + Y(KB))*COL(J);
    end
end
VL(37) = VL(1);
SY(37) = SY(1);
if IDEN == 0
    for I
                = 2:18
              = 38-I;
        ΙA
        VL(IA) = -VL(I);
        SY(IA) = SY(I);
    end
end
Y(1) = ABA;
    CALCULATION OF BASE PROFILE VELOCITY AND INCREMENT DUE TO ANGLE OF
   ATTACK
if prnt2scrn == 1
    fprintf('\n\n
                                           PROFILE CONSTANTS\n\n');
    fprintf('\t X\t\tY\t\t\tC\t\t\tD\t\tE\t\t\ DY/DPHI\n');
end
for I=1:37
           = sqrt(SY(I)^2 + (SO(I)^2)/4.);
    CC(I) = (VL(1)-VL(I)-SO(I)/2.)/D2;
    DD(I) = (SY(I) - SY(1) + (CO(I) - 1)/2)/D2;
    AAAAA = (X(I)-1)/(2*D2)+X(I)*DD(I);
    TΑ
            = -CC(I);
    TB
            = -DD(I);
    EE(I)
            = D2;
    Profile Const(I,:) = [X(I) Y(I) TA TB AAAAA SY(I)];
    if prnt2scrn == 1
        fprintf('%12.6f %12.6f %12.6f %12.6f %12.6f %12.6f\n',...
            X(I), Y(I), TA, TB, AAAAA, SY(I);
    end
end
if prnt2scrn == 1
    fprintf('\n\n NON-DIMENSIONAL VELOCITY, V=(C*cos(ALFA)+D*sin(ALFA))*(1-
/+DELTA*sqrt(X-X^2))+DELTA*E) \n')
end
응
   CALCULATION OF LIFT CURVE SLOPE AND ANGLE OF ZERO LIFT, THEORY
P1 = (1+2*SY(1))^2+4*VL(1)^2;
P = sqrt(P1);
AOL = atan(2*VL(1)/(1+2*SY(1)));
```

```
AXL = AOL*180/pi;
if prnt2scrn == 1
    fprintf(' DCL/D(ALPHA)/2PI (THEORY)=%10.6f\n ANGLE,CL=0 (THEORY) =%10.6f
DEG \n\n\n\n'...
        ,P, AXL)
end
응
   CALCULATION OF BASE PROFILE VELOCITY AND INCREMENT DUE TO ANGLE OF
용
   ATTACK AT INTERMEDIATE POINTS
if prnt2scrn == 1
    fprintf('
                                    PROFILE CONSTANTS\n\n')
    fprintf('
                                    INTERMEDIATE VALUES\n')
    fprintf('
                                      UPPER SURFACE\n')
    end
for I=1:12
    CD1 = 0.0;
    CD2 = 0.0;
    CD3 = 0.0;
    CD4 = 0.0;
    for J=1:17
       JC = 37-J;
       YT = Y(J+1) - Y(JC);
       YC = Y(J+1) + Y(JC);
       CD1 = CD1+YT*Z1(I,J);
       CD2 = CD2 + YC \times Z2 (I, J);
       CD3 = CD3 + YT * Z3 (I, J);
       CD4 = CD4 + YC \times Z4 (I, J);
    end
    DYU
          = CD3+CD4;
           = CD3-CD4;
    DYL
           = CD1+CD2;
    CTU
    CTL
           = -CD1+CD2;
           = SNT(I) *SNT(I) *.25;
    TA
           = .5*(1.+CNT(I));
   ANTRP(I) = DYL;
   E1(1,I) = sqrt(DYU*DYU+TA);
   E1(2,I) = sqrt(DYL*DYL+TA);
   C1(1,I) = (VL(1)-CTU-SNT(I)*.5)/E1(1,I);
    C1(2,I) = (VL(1)-CTL+SNT(I)*.5)/E1(2,I);
    D1(1,I) = (DYU-SY(1)-TB)/E1(1,I);
    D1(2,I) = (DYL-SY(1)-TB)/E1(2,I);
           = (XA(I)-1.)/(2.*E1(1,I))+XA(I)*D1(1,I);
    TΑ
           = -C1(1, I);
    TB
           = -D1(1,I);
    Profile Const INT U(I,:) = [XA(I) TA TB AAAAA DYU];
    if prnt2scrn == 1
        fprintf('%12.6f%12.6f%12.6f%12.6f\n',...
           XA(I), TA, TB, AAAAA, DYU)
    end
end
if prnt2scrn == 1
    fprintf('\n\n
                                          LOWER SURFACE\n')
    end
```

```
for I=1:12
   AAAAA = (XA(I)-1.)/(2.*E1(2,I))+XA(I)*D1(2,I);
           = -C1(2, I);
          = -D1(2,I);
   TB
   Profile Const INT L(I,:) = [XA(I) TA TB AAAAA ANTRP(I)];
   if prnt2scrn == 1
       fprintf('%12.6f', XA(I), TA, TB, AAAAA, ANTRP(I))
       fprintf('\n')
   end
end
if ICL == 0
   AOLE = AOLE+AWK;
   AXL
           = AOLE*.017453293;
end
  CALCULATION OF THEORETICAL AND DISTORTED PRESSURE DISTRIBUTION
if IPMIN > 0
   if prnt2scrn == 1
       fprintf('\n\n
                                            MINIMUM PRESSURES\n')
                              ETA = %8.6f,',ETA)
       fprintf('
       fprintf('
                  ALFA(CL=0) = %10.6f\n\n',AOLE)
       fprintf(' ALFA CL
                                         CP MIN
                                                     MAX VELOC
CM(X=0.25) \n')
   end
end
for I=1:JA
    if IPMIN == 0
       if prnt2scrn==1
           fprintf('\n\n
                                              PRESSURE DISTRIBUTION\n')
       end
       if IDEN == 0
           if prnt2scrn == 1
               fprintf('
                                               SYMMETRICAL PROFILE\n\n')
           end
       elseif IDEN > 0
           if prnt2scrn == 1
               fprintf('
                                            NON-SYMMETRICAL PROFILE\n\n')
           end
       end
       if prnt2scrn == 1
           fprintf(' ALFA CL DELTA sin(ALFA) LIFT
SLOPE ALFA, CL=0\n')
       end
   end
   DMALFA = ALFA(I) + AWK;
   ANG = DMALFA*pi/180;
    if ICL ~= 0
       CL = CLE(I);
    elseif ICL == 0
       CL = 2*pi*ETA*(ANG-AXL);
   end
          = ANG - AOL - atan(CL/sqrt(39.478418*P1 - CL^2));
   DEL
   SA
           = sin(ANG);
   SA = sin(ANG);
CA = cos(ANG);
   if IPMIN == 0
```

```
if prnt2scrn == 1
            fprintf('%12.6f', DMALFA, CL, DEL, SA, ETA, AOLE)
            fprintf('\n')
                          X POTNL VELOC VISC INCRM VISC VELOC
            fprintf('\n\n
POTNL P/O
            VISC P/O\n')
       end
    end
   APG
           = ANG-DEL;
    CAP
           = \cos(APG);
    SAP
            = sin(APG);
    CLINT
           = 0.0;
   CDINT
           = 0.0;
    CMXINT = 0.0;
    CMYINT = 0.0;
    SIGMA
           = 0.0;
    for J=1:37
        APG
              = ANG-DEL*X(J);
       CAV
               = cos(APG);
       SAV
              = sin(APG);
              = abs(CC(J)*CA+DD(J)*SA);
       VELP
       VELV
              = (1-DEL*SO(J)/2.)*((CC(J)-VL(1)/EE(J))*CAV+(DD(J)...
           +(.5+SY(1))/EE(J))*SAV+VL(1)/EE(J)*CAP-(.5+SY(1))/EE(J)*SAP);
       VELV
              = abs(VELV);
       ANCR
               = VELV-VELP;
       PRESP = 1-(VELP)^2;
       PRESV = 1-(VELV)^2;
              = pi/AN*PRESV;
               = ABD*SO(J)/2;
       CMXINT = CMXINT+ABC* (X(J) - .25);
        if IPMIN == 0
            CLINT = CLINT-ABC;
            if (J-1) > 0
                      = ABD*SY(J);
                ABD
                CDINT = CDINT-ABD;
                if (J-37) < 0
                    CMYINT = CMYINT-ABD*Y(J);
                end
            end
            Press Dist(J,:) = [X(J) VELP ANCR VELV PRESP PRESV];
            if prnt2scrn == 1
                fprintf('%12.6f', X(J), VELP, ANCR, VELV, PRESP, PRESV)
                fprintf('\n')
            end
        elseif IPMIN > 0
            if (SIGMA-PRESV) >=0
                SIGMA = PRESV;
                XMIN = X(J);
                VMAX
                      = VELV;
            end
        end
        if (37-J) == 0
            if IPMIN == 0
                if prnt2scrn == 1
                    fprintf('\nINTEGRATED CN=%10.6f\n',CLINT)
   4508 PRINT 17, CLINT
                    fprintf('INTEGRATED CC=%10.6f\n',CDINT)
        PRINT 18, CDINT
```

```
fprintf('INTEGRATED CM(X)=%10.6f, CW ABT
                             PRINT 19, CMXINT
X=0.25 \n', CMXINT)
                    %
                   fprintf('INTEGRATED CM(Y)=%10.6f, CW ABT Y=0\n', CMYINT)
      PRINT 20, CMYINT
9
엉
   CALCULATION AT INTERMEDIATE POINTS
                   fprintf('\n\n
                                                     PRESSURE
                            PRINT 8
DISTRIBUTION\n\n')
                     용
          fprintf(' ALFA CALFA, CL=0\n') % PRINT 12
                                      CL
                                                      DELTA sin(ALFA)
LIFT SLOPE
                   fprintf('%12.6f', DMALFA,CL,DEL,SA,ETA,AOLE)
% PRINT 2, DMALFA, CL, DEL, SA, ETA, AOLE
                   fprintf('\n\n
                                                       INTERMEDIATE
VALUES\n')
                   % PRINT 14
응 C
응 C
       UPPER SURFACE NOSE VELOCITY
                  fprintf('\n
                                                       UPPER SURFACE \n')
            fprintf(' X POTNL VELOC VISC INCRM VISC VELOC
VISC P/Q \n') % PRINT 13
      PRINT 15
POTNL P/O
               end
           end
           for K=1:12
               APG = ANG-DEL*XA(K);
               CAV
                     = cos(APG);
                     = sin(APG);
               SAV
               VELP = abs(C1(1,K)*CA+D1(1,K)*SA);
               VELV = abs ((1.-DEL*SNT(K)/2.)*((C1(1,K)-VL(1)...
                   /E1(1,K))*CAV+(D1(1,K)+(.5+SY(1))/E1(1,K))*SAV+...
                   (VL(1) *CAP-(.5+SY(1))*SAP)/E1(1,K)));
                      = VELV-VELP;
               ANCR
               PRESP = 1 - (VELP)^2;
               PRESV = 1 - (VELV)^2;
               if IPMIN > 0
                   if (SIGMA-PRESV) >= 0
                       SIGMA = PRESV;
                       XMIN = XA(K);
                       VMAX = VELV;
                   end
               elseif IPMIN == 0
                   Press Dist INT U(K,:) = [XA(K) VELP ANCR VELV PRESP]
PRESV];
                   if prnt2scrn == 1
                       fprintf('%12.6f', XA(K), VELP, ANCR, VELV, PRESP, PRESV)
                       fprintf('\n')
                   end
               end
           end
응
    LOWER SURFACE NOSE VELOCITY
응
           if IPMIN == 0
               if prnt2scrn == 1
                   fprintf('\n
                                                 LOWER SURFACE\n')
```

```
fprintf(' X POTNL VELOC VISC INCRM VISC VELOC
POTNL P/Q
            VISC P/Q\n')
                end
            end
            for K=1:12
                APG
                       = ANG-DEL*XA(K);
                CAV
                       = cos(APG);
                SAV
                       = sin(APG);
                       = abs(C1(2,K)*CA+D1(2,K)*SA);
                VELP
                     = abs((1.+DEL*SNT(K)/2.)*((C1(2,K)-VL(1)...
                VELV
                    /E1(2,K))*CAV+(D1(2,K)+(.5+SY(1))/E1(2,K))...
                    *SAV+(VL(1)*CAP-(.5+SY(1))*SAP)/E1(2,K)));
                ANCR
                       = VELV - VELP;
                PRESP = 1-VELP^2;
                PRESV = 1-VELV^2;
                if IPMIN > 0
                    if (SIGMA-PRESV) >= 0
                        SIGMA = PRESV;
                               = XA(K);
                        XMIN
                        VMAX
                               = VELV;
                    end
                elseif IPMIN == 0
                    Press Dist INT L(K,:) = [XA(K) VELP ANCR VELV PRESP]
PRESV1;
                    if prnt2scrn == 1
                        fprintf('%12.6f', XA(K), VELP, ANCR, VELV, PRESP, PRESV)
                        fprintf('\n')
                    end
                end
            end
            if IPMIN > 0
                Min Press(I,:) = [DMALFA CL SIGMA VMAX XMIN CMXINT];
                if prnt2scrn == 1
                    fprintf('%12.6f', DMALFA, CL, SIGMA, VMAX, XMIN, CMXINT)
                    fprintf('\n')
                end
            end
        end
    end
end
   END OF BROCKETT'S CODE
% Combine Profile Constant Data in Order
i = 1;
j = length(Profile Const INT U);
step = 1;
while j>0 && i<length(Profile Const)</pre>
    A = Profile Const(i, 1);
    B = Profile_Const_INT_U(j,1);
    if A > B
        Profile Comb(step,:) = Profile Const(i,:);
        i = i + 1;
        step = step + 1;
    elseif B > A
```

```
Profile Comb(step,:) = [Profile Const INT U(j,1) 0
Profile Const INT U(j,2:end)];
        j = j -1;
        step = step + 1;
    end
end
\dot{1} = 1;
while j <= length(Profile Const INT L) || i <= length(Profile Const)</pre>
    if B < A && j <= length(Profile Const INT L)</pre>
        Profile Comb(step,:) = [Profile Const INT L(j,1) 0
Profile_Const_INT_L(j,2:end)];
        j = j + 1;
        if j <= length(Profile Const INT L)</pre>
            B = Profile Const INT L(j,1);
        end
        step = step + 1;
    elseif A < B || j > length(Profile Const INT L)
        Profile Comb(step,:) = Profile Const(i,:);
        i = i + 1;
        if i <= length(Profile Const)</pre>
            A = Profile Const(i, 1);
        end
        step = step + 1;
    end
end
    Combine Pressure Distribution Data in Order
if IPMIN == 0
    i = 1;
    j = length(Press Dist INT U);
    step = 1;
    while j>0 && i<length(Press Dist)
        A = Press Dist(i,1);
        B = Press Dist INT_U(j,1);
        if A > B
            Press Comb(step,:) = Press Dist(i,:);
            i = i + 1;
            step = step + 1;
        elseif B > A
            Press Comb(step,:) = Press Dist INT U(j,:);
             j = j -1;
            step = step + 1;
        end
    end
    j = 1;
    while j <= length(Press Dist INT L) || i <= length(Press Dist)</pre>
        if B < A && j <= length(Press Dist INT L)</pre>
            Press_Comb(step,:) = Press_Dist_INT_L(j,:);
             j = j + 1;
            if j <= length(Press Dist INT L)</pre>
                 B = Press Dist INT L(j,1);
            end
            step = step + 1;
        elseif A < B || j > length(Press Dist INT L)
            Press Comb(step,:) = Press Dist(i,:);
            i = i + 1;
            if i <= length(Press Dist)</pre>
```

```
A = Press Dist(i,1);
           end
           step = step + 1;
       end
    end
end
% Calls XFOIL calculate pressure distribution and imports data
cmd = ['xfoil.exe LOAD trefxy OPER ALFA ', num2str(ALFA),...
   ' OPER CPWR CPX'];
system(cmd);
fid = fopen('CPX');
xfoil data in = textscan(fid, '%f64 %f64', 'headerlines', 1);
fclose(fid);
xfoil cp = xfoil_data_in{2};
   Plots Data from Brockett and Xfoil on same graph as Trefftz.
open('Trefftz.fig')
plot(Press Comb(:,1), -Press Comb(:,5), 'ro-')
plot(Press_Comb(:,1), -Press_Comb(:,6), 'r.-')
plot(xfoil_x, -xfoil_cp, 'bx-')
legend('Conformal Transformation','Brockett Method (Inviscid)',...
    'Brockett Method (Viscid)', 'XFOIL Calc')
savefile = ['K-T, alfa=', num2str(ALFA),];
saveas(gcf, savefile);
```

Appendix C: Brockett.m Variable Descriptions

Function variables are specified as those variable that instruct the MATLAB version of Brockett's work how to process the data, which data will be input, and how the data is formatted. The following paragraph provide a brief description of these variables, and how they are used in Brockett.m

JA: Specifies the number of angles of attack that will be calculated. Although not specified directly by the user, the scripts in Appendix C: calculate this value based on the number of inputs for the ALFA vector. It is recommended that multiple angles of attack be processed individually, since output for multiple angles is only printed to the screen.

KA: This variable was used originally by Brockett for processing multiple jobs. This functionality is not used in the MATLAB version. Separate jobs are specified by the appropriate script file (i.e. ARB_IN.m, REQ_IN.m, etc), in which the use specifies the job parameters. The job specification script is specified by the "run" command in the beginning of Brockett.m.

IPMIN: Specifies whether or not minimum pressure distribution data will be reported. Should normally be set to 0. Plotting and screen output will not be available if minimum pressure data is not calculated. If screen output is not desired, use the 'print2scr' variable below.

ALFA: Vector of angles of attack to be calculated. Normally a single value. Note: If multiple angles of attack are specified, output for each angle of attack will be only to screen in tabular format. Plots for each angle of attack will not be generated.

IDEN: Specifies whether input data points are for a symmetric foil shape. Symmetric data is designated by IDEN = 0, or non-symmetric IDEN = 1. Symmetric data can either be in the format of offsets, or camber and thickness data. **IPM:** IPM = 0 specifies that ordinate information will be input at required locations. Required locations are specified by:

$$x_{m} = \frac{1}{2} \left(1 + \cos \frac{m \cdot \pi}{18} \right), m = 0...18$$
$$x_{36-m} = x_{m}$$

When input at required station is specified, only the upper surface, Y_0 through Y_{17} , are specified for symmetric foils. All others, Y_0 through Y_{35} are specified. If IPM = 1 (arbitray input locations), user must define the number of locations (NX) that will be input.

ICL: ICL = 1 if experimental lift coefficient is specified rather that angle of zero lift (AOLE) and lift-slope curve coefficient (ETA). If ICL = 0, AOLE and ETA must be set to zero. Otherwise, if ICL = 0, AOLE and ETA must be specified. AOLE and ETA values do not affect inviscid calculation, and are only used for the empirical modification to account for viscous effects. If unknown, may be set to 0 for inviscid calculations.

CLE: If ICL = 1, user must specify experimental lift coefficients corresponding to input angles of attack (ALFA). ALFA and CLE vector must be of equal length.

ILK: ILK = 0 specifies that offsets will be input in X, Y format. CC is vector of x-values, and Y is vector of corresponding y-offset values. For symmetric foils, give only upper surface from trailing edge to nose. Last point must be (0, 0). For non-symmetric foils, must specify XN and YN which are x and y ordinates of nose location. Sample scripts locate this point automatically from input vectors CC and Y. Order for non-symmetric foils must be from trailing edge (1, X.X), along upper surface to (XN, YN), and back to trailing edge (1.0, X.X).

ILK = 1 specifies that foil surface locations will be specified by thickness ratio (TAO), camber (F), and leading edge radius (RO). Input required is x-location (AT_in), thickness value (YT_in), camber value (YC_in), and camberline slope

(YCP_in). AT_in, YT_in, YC_in, and YCP_in start at trailing edge(x=1) and go to leading edge (x=0), and are of length NX.

foil_name: User specified foil designation. Used for plot legend and/or titles.

Appendix D: Sample Input Scripts for Brockett.m.

REQD IN.m

```
Written by Chris Peterson
This file inputs data to be processed by MATLABs version of
  Brockett's Thesis program. The data is from fig 3b, pg 67.
% Format is for input at required stations, angle of attack specified
ALFA
       = 4.09;
                      %ANGLE OF ATTACK
      = length(ALFA); %NUMBER OF ANGLES
IPMIN = 0; %0:Report Data, 1: No Data
                    %0:SYMM, 1:NONSYMM
%0:STD INPUT LOCATIONS, 1:ARBITRARY STATIONS
IDEN = 0;
IPM = 0;
ICL
      = 0;
                      %0:USE ALPHA, 1: USE INPUT CLE
AOLE = 0;
                      %Experimental angle of zero lift
ETA = 0.959; %Lift curve-slope coeff
foil name = 'RAE-101,00-10';
% Y in is array of ordinates, trailing edge to leading edge along upper
% surface, then leading edge to trailing edge along lower surface. Only
  Yn n=0->17 for symmetric foils. All others n=0->35
Y \text{ in} = [0 .00068 .0027 .00599 .01046 .01597]
                                                         .02236...
    .029345 .03636 .04267 .047445 .04985 .04885...
.04475 .038405 .03034 .02093 .01071 ];
% Formats Y for both surfaces
Y = [Y in 0 fliplr(-Y in)];
%Plot input
figure(1)
axis equal;
hold on;
plot((1+cos(0:2*pi/36:2*pi))/2, Y, 'g.')
plot((1+cos(0:2*pi/36:2*pi))/2, Y, 'g')
legend('Input to Brockett')
title(foil name)
NX = 0;
                  %Req'd to set variable EE (used in Arb Input)
```

ARB IN.m

```
Written by Chris Peterson
   This file inputs data to be processed by the MATLAB version of
  Brockett's Thesis program. The data is from fig 4b, pg 69.
  Format is for arbitrary input stations, non-symmetrical,
   lift coefficient specified
IPMIN = 0;
                       %0:Report Data, 1: No Data
IDEN = 1;
                       %0:SYMM, 1:NONSYMM
      = 1;
IPM
                       %0:STD INPUT LOCATIONS, 1:ARBITRARY STATIONS
      = 1;
                       %0:USE ALPHA, 1: USE INPUT CLE
 CLE = [-0.14 .15 .44 .73 .97 1.16 1.26 1.34 1.11]; %Lift Coeff
 ETA = 0;
 AOLE = 0;
     = 0;
                       %0:INPUT STA X,Y 1:INPUT TAU, RHO, RHO
ALFA = [-7.6 -4.5 -1.5 1.5 4.7 8.0 9.7 11.4 16.2]; %ANGLE OF ATTACK
      = length(ALFA); %NUMBER OF ANGLES
foil name = 'CLARK Y, NACA RPT 460';
% Coordinates. Must have same number on upper surface as lower surface.
% For symmetrical foil, give only upper surface (last point 0,0 for
% symmetrical foils)
CC = [1 .992404 .95 .9 .8 .7 .6 .5 .4 .3 .2 .15 .1 .075 .05 .025 .0125...
    0 .0125 .025 .05 .075 .1 .15 .2 .3 .4 .5 .6 .7 .8 .9 .95 .992404 1];
Y = [.0006 \quad .0027 \quad .0144 \quad .0273 \quad .0515 \quad .0728 \quad .0907...
    .1043 .1131 .1162 .1126 .1057 .0950 .0873 .077
.0637 .0532 .0354 .0180 .0136 .0085 .0053...
.0033 .0008 -.0005 -.0006 -.0006 -.0006...
    -.0006 -.0006 -.0006 -.0006 -.0006];
[XN, indx] = min(CC);
YN = Y(indx);
NX = length(CC);
%Plot input
figure()
axis equal;
hold on;
plot(CC,Y, 'k.', CC,Y, 'k');
legend('Input to Brockett')
title(foil name)
```

KT IN.m

```
Written by Chris Peterson
   This file inputs data to be processed by the MATLAB version of
% Brockett's Thesis code (Brockett.m). This file reads input generated
% by the Karman-Trefftz foil Conformal Transformation script
(CnfrmlTrans.m).
IPMIN = 0;
                      %0:Report Data, 1: No Data
IDEN = 1;
                       %0:SYMM, 1:NONSYMM
   IPM
IPM ICL = 0;
if ICL == 0
elseif ICL ==1
    CLE = 0; %Experimental lift coefficient

AOLE = 0; %AOLE and ETA must be set to 0 if ICL = 1

ETA = 0; %AOLE and ETA must be set to 0 if ICL = 1
end
                       %0:INPUT STA X,Y 1:INPUT TAU, RHO, RHO
      = 0;
ILK
foil name = 'Karman-Trefftz';
load('x output.mat', '-mat'); %Opens data generated by CnfrmlTrans.m
CC
        = x spl;
                       %Read in x data from K-T foil ordinates
        = y spl; %Read in y data from K-T foil ordinates
[XN, xn ind] = min(CC); %Find nose x location and index
NX = length(CC); %NUMBER OF STATIONS
YN = Y(xn_ind); %Specifies nose y location
ALFA = alpha_deg; %ANGLE OF ATTACK (FOR DESIRED PRESSURE DIST)
      = length(ALFA); %NUMBER OF ANGLES
JA
%Plot input
figure()
axis equal;
hold on;
plot(CC,Y, 'k.', CC,Y, 'k');
legend('Input to Brockett')
title(foil name)
%Compare to Karman-Trefftz foil? 1-yes, 0-no
comp2kt = 1; %Opens previous trefftz plot and plots new data
```

Brock IN.m

```
Written by Chris Peterson
   This file inputs data to be processed by MATLAB version of
% Brockett's Thesis program. The data is from Brockett's published
% minimum pressure envelopes for NACA foils (DTMB Report 1780, pg 14).
IPMIN = 0;
                      %0:Report Data, 1: No Data
                     %0:SYMM, 1:NONSYMM
IDEN = 1;
                     %0:STD INPUT LOCATIONS, 1:ARBITRARY STATIONS
IPM
      = 1;
ICL
      = 1;
                      %0:USE ALPHA, 1: USE INPUT CLE
 ETA = 0;
 AOLE = 0;
ILK = 1;
                      %0:INPUT STA X,Y 1:INPUT TAU, RHO, RHO
ALFA = 0;
                      %ANGLE OF ATTACK
      = length(ALFA); %NUMBER OF ANGLES
foil name = 'NACA 66 (Mod), a=0.8';
TAO = 0.12;
F = 0.06;
RHO = .448;
AT in = fliplr([0\ 0.007596\ 0.030154\ 0.066987\ .116978\ .178606\ .25\ .32899
.413176...
   .5 .586824 .671010 .75 .821394 .883022 .933013 .969846 .992404 1]);
YT in = fliplr([0.0817.1608.2388.3135.3807.4363.4760.4972.4962
.4712...
    .4247 .3612 .2872 .2108 .1402 .0830 .0462 .0333]);
YC in = fliplr([0.06006.18381.33684.49874.65407.79051.89831.96994
    .98503 .92306 .81212 .63884 .42227 .23423 .09982 .02365 0]);
YCP in = fliplr([7.1485 \ 6.6001 \ 4.7712 \ 3.6751 \ 2.8681 \ 2.2096 \ 1.6350 \ 1.1071
.6001 ...
    .0914 -.4448 -1.0483 -1.8132 -3.1892 -3.7243 -3.7425 -3.5148 -3.2028...
   -3.0025]);
NX = length(AT in);
CLE = 2*pi*(1-0.83*TAO)*(deg2rad(ALFA) + 2.05*F); %Lift Coeff
nose rad = RHO*TAO^2;
comp2kt = 0;
```

Appendix E: Brockett.m Sample Output

PROFILE CONSTANTS						
X	Y	C	D	E	DY/DPHI	
1.000000	0.000000	0.000000	0.000000	0.000000	0.001998	
0.992404	0.000680	0.890691	0.026481	-0.069870	0.007287	
0.969846	0.002700	0.933471	0.096858	-0.181740	0.015520	
0.933013	0.005990	0.958680	0.186467	-0.307426	0.022185	
0.883022	0.010460	0.978938	0.279256	-0.427845	0.028863	
0.821394	0.015970	0.997232	0.380868	-0.545076	0.034145	
0.750000	0.022360	1.016276	0.490314	-0.655257	0.038834	
0.671010	0.029345	1.038498	0.615677	-0.761927	0.040635	
0.586824	0.036360	1.062833	0.761407	-0.865047	0.039076	
0.500000	0.042670	1.088804	0.937296	-0.967603	0.032369	
0.413176	0.047445	1.114413	1.150647	-1.070719	0.021690	
0.328990	0.049850	1.139587	1.422610	-1.182065	0.004568	
0.250000	0.048850	1.147911	1.771430	-1.308325	-0.015548	
0.178606	0.044750	1.144379	2.222439	-1.465799	-0.030556	
0.116978	0.038405	1.137954	2.859395	-1.696827	-0.041660	
0.066987	0.030340	1.125485	3.863976	-2.087954	-0.050476	
0.030154	0.020930	1.087640	5.708723	-2.863493	-0.056743	
0.007596	0.010710	0.945091	9.983659	-4.773447	-0.060158	
0.000000	0.000000	0.000000	17.144979	-8.056392	-0.062063	
0.007596	-0.010710	-0.945091	9.983659	-4.773447	-0.060158	
0.030154	-0.020930	-1.087640	5.708723	-2.863493	-0.056743	
0.066987	-0.030340	-1.125485	3.863976	-2.087954	-0.050476	
0.116978	-0.038405	-1.137954	2.859395	-1.696827	-0.041660	
0.178606	-0.044750	-1.144379	2.222439	-1.465799	-0.030556	
0.250000	-0.048850	-1.147911	1.771430	-1.308325	-0.015548	
0.328990	-0.049850	-1.139587	1.422610	-1.182065	0.004568	
0.413176	-0.047445	-1.114413	1.150647	-1.070719	0.021690	
0.500000	-0.042670	-1.088804	0.937296	-0.967603	0.032369	
0.586824	-0.036360	-1.062833	0.761407	-0.865047	0.039076	
0.671010	-0.029345	-1.038498	0.615677	-0.761927	0.040635	
0.750000	-0.022360	-1.016276	0.490314	-0.655257	0.038834	
0.821394	-0.015970	-0.997232	0.380868	-0.545076	0.034145	
0.883022	-0.010460	-0.978938	0.279256	-0.427845	0.028863	
0.933013	-0.005990	-0.958680	0.186467	-0.307426	0.022185	
0.969846	-0.002700	-0.933471	0.096858	-0.181740	0.015520	
0.992404	-0.000680	-0.890691	0.026481	-0.069870	0.007287	
1.000000	0.000000	0.000000	0.000000	0.000000	0.001998	

NON-DIMENSIONAL VELOCITY, V= $(C*cos(ALFA)+D*sin(ALFA))*(1-/+DELTA*sqrt(X-X^2))+DELTA*E)$ DCL/D(ALPHA)/2PI (THEORY)= 1.003996 ANGLE,CL=0 (THEORY) = 0.000000 DEG

PROFILE CONSTANTS

INTERMEDIATE VALUES UPPER SURFACE

X	C	D	E	DY/DPHI
0.000076	0.161276	16.982900	-7.981671	-0.062038
0.000305	0.313757	16.522600	-7.769532	-0.061966
0.000685	0.450819	15.831154	-7.451069	-0.061848
0.001218	0.569054	14.992649	-7.065239	-0.061689
0.001903	0.667997	14.085384	-6.648289	-0.061492
0.002739	0.749135	13.169455	-6.228004	-0.061265
0.003727	0.814891	12.284473	-5.822656	-0.061012
0.004866	0.867893	11.452912	-5.442571	-0.060740
0.006156	0.910580	10.685001	-5.092396	-0.060456
0.011852	1.005922	8.501872	-4.103034	-0.059397
0.017037	1.043992	7.345885	-3.584794	-0.058606
0.023142	1.069452	6.436098	-3.181521	-0.057745

LOWER SURFACE

X	C	D	E	DY/DPHI
0.000076	-0.161276	16.982900	-7.981671	-0.062038
0.000305	-0.313757	16.522600	-7.769532	-0.061966
0.000685	-0.450819	15.831154	-7.451069	-0.061848
0.001218	-0.569054	14.992649	-7.065239	-0.061689
0.001903	-0.667997	14.085384	-6.648289	-0.061492
0.002739	-0.749135	13.169455	-6.228004	-0.061265
0.003727	-0.814891	12.284473	-5.822656	-0.061012
0.004866	-0.867893	11.452912	-5.442571	-0.060740
0.006156	-0.910580	10.685001	-5.092396	-0.060456
0.011852	-1.005922	8.501872	-4.103034	-0.059397
0.017037	-1.043992	7.345885	-3.584794	-0.058606
0.023142	-1.069452	6.436098	-3.181521	-0.057745

PRESSURE DISTRIBUTION SYMMETRICAL PROFILE

ALFA CL DELTA sin(ALFA) LIFT SLOPE ALFA,CL=0 4.090000 0.430129 0.003146 0.071323 0.959000 0.000000

X	POTNL VELOC	VISC INCRM	VISC VELOC	POTNL P/Q	VISC P/Q
1.000000		0.000000	0.000000	1.000000	1.000000
0.992404	0.890311	-0.000269	0.890042	0.207346	0.207825
0.969846	0.938002	-0.000877	0.937124	0.120153	0.121798
0.933013	0.969537	-0.001532	0.968005	0.059997	0.062965
0.883022	0.996363	-0.002161	0.994201	0.007261	0.011564
0.821394	1.021857	-0.002763	1.019094	-0.044191	-0.038552
0.750000	1.048659	-0.003318	1.045341	-0.099685	-0.092737
0.671010	1.079765	-0.003835	1.075931	-0.165893	-0.157627
0.586824	1.114433	-0.004305	1.110128	-0.241960	-0.232384
0.500000	1.152882	-0.004731	1.148151	-0.329138	-0.318250
0.413176	1.193643	-0.005110	1.188533	-0.424784	-0.412611
0.328990	1.238150	-0.005460	1.232689	-0.533015	-0.519523
0.250000	1.271331	-0.005780	1.265552	-0.616284	-0.601622
0.178606	1.299976	-0.006129	1.293847	-0.689939	-0.674040
0.116978	1.338998	-0.006662	1.332336	-0.792914	-0.775119
0.066987	1.398210	-0.007654	1.390556	-0.954992	-0.933647
0.030154	1.492035	-0.009812	1.482224	-1.226170	-1.196987
0.007596	1.654753	-0.015487	1.639265	-1.738206	-1.687191
0.000000	1.222837	-0.025387	1.197451	-0.495331	-0.433888
0.007596		0.015110	0.245726	0.946816	0.939619
0.030154		0.009398	0.687103	0.540716	0.527890
0.066987	0.847027	0.007264	0.854291	0.282546	0.270187
0.116978	0.931114	0.006319	0.937434	0.133026	0.121218
0.178606		0.005850	0.988803	0.033804	0.022270
0.250000		0.005575	1.024218	-0.037633	-0.049022
0.328990		0.005338	1.040558	-0.071679	-0.082760
0.413176		0.005071	1.034578	-0.059885	-0.070351
0.500000		0.004772	1.023952	-0.038728	-0.048479
0.586824		0.004421	1.010241	-0.011675	-0.020588
0.671010		0.004020	0.995961	0.016053	0.008062
0.750000		0.003565	0.982282	0.042113	0.035122
0.821394		0.003062	0.970590	0.063891	0.057956
0.883022		0.002504	0.959032	0.085055	0.080258
0.933013		0.001906	0.944844	0.110867	0.107269
0.969846		0.001268	0.925453	0.145882	0.143537
0.992404		0.000656	0.887190	0.214058	0.212894
1.000000	0.000000	0.000000	0.000000	1.000000	1.000000

INTEGRATED CN= 0.425593 INTEGRATED CC= -0.030299 INTEGRATED CM(X)= 0.002425, CW ABT X=0.25 INTEGRATED CM(Y)= -0.001200, CW ABT Y=0

PRESSURE DISTRIBUTION

ALFA CL DELTA sin(ALFA) LIFT SLOPE ALFA,CL=0 4.090000 0.430129 0.003146 0.071323 0.959000 0.000000

INTERMEDIATE VALUES

0.023142

0.607684

	UPPER SURFA	CE			
X	POTNL VELOC	VISC INCRM	VISC VELOC	POTNL P/Q	VISC P/Q
0.000076	1.372143	-0.025188	1.346954	-0.882775	-0.814286
0.000305	1.491405	-0.024563	1.466842	-1.224289	-1.151624
0.000685	1.578802	-0.023607	1.555195	-1.492615	-1.418630
0.001218	1.636931	-0.022440	1.614491	-1.679543	-1.606580
0.001903	1.670913	-0.021175	1.649737	-1.791949	-1.721633
0.002739	1.686517	-0.019899	1.666619	-1.844340	-1.777618
0.003727	1.688985	-0.018667	1.670318	-1.852671	-1.789962
0.004866	1.682543	-0.017513	1.665029	-1.830950	-1.772323
0.006156	1.670351	-0.016452	1.653899	-1.790074	-1.735383
0.011852	1.609742	-0.013469	1.596273	-1.591269	-1.548088
0.017037	1.565266	-0.011923	1.553343	-1.450057	-1.412873
0.023142	1.525772	-0.010735	1.515037	-1.327982	-1.295338
	LOWER SURF.	ACE			
X	POTNL VELOC	VISC INCRM	VISC VELOC	POTNL P/Q	VISC P/Q
0.000076	1.050412	-0.025123	1.025289	-0.103366	-0.051217
0.000305	0.865489	-0.024437	0.841053	0.250928	0.292630
0.000685	0.679460	-0.023425	0.656035	0.538334	0.569618
0.001218	0.501721	-0.022211	0.479510	0.748276	0.770070
0.001903	0.338321	-0.020906	0.317415	0.885539	0.899248
0.002739	0.192062	-0.019597	0.172465	0.963112	0.970256
0.003727	0.063355	-0.018340	0.045015	0.995986	0.997974
0.004866	0.048822	0.017165	0.065988	0.997616	0.995646
0.006156	0.146171	0.016087	0.162258	0.978634	0.973672
0.011852	0.396978	0.013070	0.410048	0.842409	0.831861
0.017037	0.517400	0.011514	0.528914	0.732298	0.720250

0.010321

0.618006

0.630720

0.618069

Appendix F: Modified XFOIL User Guide.

Appendix G: Instruction for compiling modified XFOIL Code

This appendix is intended to provide basic instructions on how to obtain and compile the source code for the modified version of XFOIL used for this project. The original source code for XFOIL as released by Mark Drela can be obtained at http://web.mit.edu/drela/Public/web/xfoil/. Version 6.96 was used for this project. In order to compile the official release version of XFOIL, it is recommended to follow the instructions that are contained in the README that is included with the *.tar files containing the source code. Since this project was conducted on a Win32 based PC, the following recommendations are provided based on personal experience while trying to compile the source code in Windows XP

- (1) Download and install Cygwin, available at http://www.cygwin.com/. Cygwin is a Linux-like environment for Windows. It consists of two parts:
- A DLL (cygwin1.dll) which acts as a Linux API emulation layer providing substantial Linux API functionality.
- A collection of tools which provide Linux look and feel.

Cygwin allows native Linux applications to be run on Windows machines, if they are rebuilt from their original source code using Cygwin. Specifically, the original source code for XFOIL made use of X11 window tools that are not normally available in Windows (Unix like plotting). The use of Cygwin was a fix to this.

Once running the Cygwin setup program, and under the "Select Packages Screen", complete the following actions prior to clicking the "Next" button:

- -Under the "Devel" pull-down menu, select the "gcc-g77: Fortran Package" for installation by click the "Skip" item on the left column. This action selects the current version to include in the installation. This action will also select add-on packages required for installation.
- -Under the "Devel" pull-down menu, select the "make: The GNU version of the 'make' utility" for installation by click the "Skip" item on the left column.

- -Under the "X11" pull-down menu, select the "xorg-x11-base: Cygwin/X base" package for installation, and the "xorg-x11-devel: Cygwin/X headers and import libraries". Associated add-ons will also be automatically selected.
- (2) A basic guide on installation these items can be found at http://www2.warwick.ac.uk/fac/sci/moac/currentstudents/peter_cock/cygwin/. Although not an official reference, the instructions and guidance provided here were found to be useful.
- (3) Download and intall MinGW. Instructions can be found by following the downloand link on the http://www.mingw.org/ web page. When running the installation program, ensure that the seletion box for 'g77 compilier' is checked under "Select Components to Install:", and continue with the installation process.
- (5) Add the following lines to the file "c:\cygwin\etc\bash.bashrc"

```
PATH=/cygdrive/c/mingw/bin:$PATH export PATH
```

These statements place the /Mingw/bin ahead of /Cygwin/bin in the path statement for the Cygwin environment, ensuring that Mingw executables for gcc.exe and g77.exe are used, rather than the Cygwin versions. This prevents an error when XFOIL is run outside of the Cygwin environment, and eliminates the error when cygwin1.dll is not present.

In order to test the above step, start Cywin and type which gcc at the Cygwin \$ prompt. The response should be /cygdrive/c/mingw/bin/gcc. If the response is /usr/bin/gcc, then the above steps were not completed properly.

(4) Download original source code from:

http://web.mit.edu/drela/Public/web/xfoil/xfoil6.96.zip

(5) Unzip xfoil6.96.zip to a working directory (i.e. C:\XFOIL\).

(6) Replace the following files in the C:\XFOIL\SRC\ directory with the files modified by the author as part of this project:

```
blplot.f
                         polplt.f
                                                  xoper.f
dplot.f
                         xfoil.f
                                                  xplots.f
qui.f
                         XFOIL.INC
                                                  xqdes.f
modify.f
                         xqdes.f
                                                  xtcam.f
plutil.f
                         xgeom.f
pntops.f
                         xmdes.f
```

(7) Place Makefile in the C:\XFOIL\SRC\ directory. Details of this file are provided below.

Makefile

```
#************
# Makefile for XFOIL V6.93 programs
# H.Youngren 4/24/01
# M.Drela
#*************
# Modified by Chris Peterson to generate modified version
# of XFOIL that removes menus and plots, and executes from
# command prompt.
#*************
SHELL = sh
#BINDIR = $(HOME)/bin/
BINDIR = .
PROGS = xfoil
# pplot pxplot
SRC = ../src
OSRC = ../osrc
XFOILOBJ = xfoil.o xpanel.o xoper.o xtcam.o xgdes.o xqdes.o \
xsolve.o xbl.o xblsys.o xpol.o xplots.o pntops.o xgeom.o xutils.o modify.o \
blplot.o polplt.o aread.o naca.o spline.o plutil.o iopol.o gui.o sort.o \
dplot.o profil.o
#PPLOTOBJ = pplot.o polplt.o sort.o iopol.o
#PXPLOTOBJ = pxplot.o plutil.o gui.o
XUTILOBJ = userio.o
FTNLIB =
```

```
##-----
OSOBJ = frplot0.0
# Use this for individual TS-wave frequency plotting
# OSOBJ = frplot.o ntcalc.o osmap.o getosfile.o
##-----
# PLTOBJ = ../plotlib/libPlt.a
# Use this if you have a copy of the plotlib as a system library
# PLTOBJ = -1Plt
# The extra location arg here is for Linux which places X libs in /usr/X11R6
# PLTLIB = -L/usr/X11R6/lib -lX11
### Default compilers and flags
### FFLOPT used for xsolve.f
FC = g77
FFLAGS = -0
FFLOPT = -0
INSTALLCMD = install -s
CC = qcc
CFLAGS = -O -DUNDERSCORE
##-----
# Uncomment flags for desired machine...
##-----
### DEC Alpha with OSF and DEC f77/f90 compiler
\#FC = f77
\#FFLAGS = -fast -04 -tune host
\#FFLOPT = -fast -04 -tune host
\#FFLOPT = -fast -05 - tune host - unroll 3
# Debug flags
\#FFLAGS = -00 -q
\#FFLOPT = -fast -04 -tune host
##-----
### SGI setup
\#FC = f77
\#FFLAGS = -02 - static
\#FFLOPT = -02 - static
##-----
## Uncomment for RS/6000
\#FFLAGS = -O - qextname
\#FFLOPT = -O - qextname
##-----
## Uncomment for HP-9000
\#FFLAGS = -O + ppu
\#FFLOPT = -O + ppu
\#FTNLIB = -U77
##-----
### Absoft Linux f77
\#FC = f77
```

```
\#FFLAGS = -0 - f - s - W - B108 - N34
\#FFLOPT = -O -f -s -W -B108 -N34
##-----
### f2c/gcc compiler driver
\#FC = fort77
#FFLAGS = -02 -fomit-frame-pointer
\#FFLOPT = -02 - fomit-frame-pointer
##-----
### GNU q77
\#FC = q77
#FFLAGS = -03 -fomit-frame-pointer
#FFLOPT = -03 -fomit-frame-pointer
# Debug flags (symbols, array bounds)
\#FC = q77
\#FFLAGS = -g -OO -C
##-----
### Intel Fortran Compiler
#FC = ifort
\#FFLAGS = -0
#FFLOPT = -0
#FTNLIB = -Vaxlib /usr/lib/C-ctype.o /usr/lib/C name.o /usr/lib/ctype-info.o
#FTNLIB = -Vaxlib
#FTNLIB = -i dynamic
##-----
### Double precision option
\#FFLAGS = -0 - r8
\#FFLOPT = -0 - r8
#PLTOBJ = ../plotlib/libPltDP.a
all: $(PROGS)
install:
     $(INSTALLCMD) $(PROGS) $(BINDIR)
clean:
     -/bin/rm $(PROGS)
     -/bin/rm $(XFOILOBJ) $(XUTILOBJ) $(OSOBJ) $(PPLOTOBJ) $(PXPLOTOBJ)
     -/bin/rm *.o
xfoil: $(XFOILOBJ) $(XUTILOBJ) $(OSOBJ)
     $(FC) -o xfoil $(XFOILOBJ) $(XUTILOBJ) $(OSOBJ) $(PLTOBJ) $(PLTLIB)
$(FTNLIB)
#pxplot:
         $(PXPLOTOBJ) $(XUTILOBJ)
    $(FC) -o pxplot $(PXPLOTOBJ) $(XUTILOBJ) $(PLTOBJ) $(PLTLIB) $(FTNLIB)
#pplot:
          $(PPLOTOBJ) $(XUTILOBJ)
# $(FC) -o pplot $(PPLOTOBJ) $(XUTILOBJ) $(PLTOBJ) $(PLTLIB) $(FTNLIB)
xfoil.o: $(SRC)/xfoil.f $(SRC)/XFOIL.INC
     $(FC) -c $(FFLAGS) $(SRC)/xfoil.f
xpanel.o: $(SRC)/xpanel.f $(SRC)/XFOIL.INC
```

```
$(FC) -c $(FFLOPT) $(SRC)/xpanel.f
xoper.o: $(SRC)/xoper.f $(SRC)/XFOIL.INC
      $(FC) -c $(FFLAGS) $(SRC)/xoper.f
xsolve.o: $(SRC)/xsolve.f $(SRC)/XFOIL.INC
      (FC) -c (FFLOPT) (SRC)/xsolve.f
dplot.o: $(SRC)/dplot.f $(SRC)/XFOIL.INC
      $(FC) -c $(FFLOPT) $(SRC)/dplot.f
xtcam.o: $(SRC)/xtcam.f $(SRC)/XFOIL.INC $(SRC)/XDES.INC
     $(FC) -c $(FFLAGS) $(SRC)/xtcam.f
xgdes.o: $(SRC)/xgdes.f $(SRC)/XFOIL.INC $(SRC)/XDES.INC
     $(FC) -c $(FFLAGS) $(SRC)/xgdes.f
xqdes.o: $(SRC)/xqdes.f $(SRC)/XFOIL.INC $(SRC)/XDES.INC
      $(FC) -c $(FFLAGS) $(SRC)/xqdes.f
xmdes.o: $(SRC)/xmdes.f $(SRC)/XFOIL.INC $(SRC)/XDES.INC $(SRC)/CIRCLE.INC
      $(FC) -c $(FFLAGS) $(SRC)/xmdes.f
xbl.o: $(SRC)/xbl.f
                        $(SRC)/XFOIL.INC $(SRC)/XBL.INC
      (FC) -c (FFLAGS) (SRC)/xbl.f
xblsys.o: $(SRC)/xblsys.f
                                          $(SRC)/XBL.INC
     $(FC) -c $(FFLAGS) $(SRC)/xblsys.f
xplots.o: $(SRC)/xplots.f $(SRC)/XFOIL.INC
     $(FC) -c $(FFLAGS) $(SRC)/xplots.f
pntops.o: $(SRC)/pntops.f $(SRC)/XFOIL.INC $(SRC)/XDES.INC
     $(FC) -c $(FFLAGS) $(SRC)/pntops.f
blplot.o: $(SRC)/blplot.f $(SRC)/XFOIL.INC
     $(FC) -c $(FFLAGS) $(SRC)/blplot.f
xpol.o: $(SRC)/xpol.f
                         $(SRC)/XFOIL.INC
     (FC) -c (FFLAGS) (SRC)/xpol.f
xgeom.o: $(SRC)/xgeom.f
      $(FC) -c $(FFLAGS) $(SRC)/xgeom.f
xutils.o: $(SRC)/xutils.f
      $(FC) -c $(FFLAGS) $(SRC)/xutils.f
modify.o: $(SRC)/modify.f
      $(FC) -c $(FFLAGS) $(SRC)/modify.f
aread.o: $(SRC)/aread.f
     $(FC) -c $(FFLAGS) $(SRC)/aread.f
naca.o: $(SRC)/naca.f
     $(FC) -c $(FFLAGS)
                         $(SRC)/naca.f
plutil.o: $(SRC)/plutil.f
     $(FC) -c $(FFLAGS)
                         $(SRC)/plutil.f
userio.o: $(SRC)/userio.f
      $(FC) -c $(FFLAGS) $(SRC)/userio.f
gui.o: $(SRC)/gui.f
      $(FC) -c $(FFLAGS)
                         $(SRC)/qui.f
spline.o: $(SRC)/spline.f
      $(FC) -c $(FFLAGS) $(SRC)/spline.f
sort.o: $(SRC)/sort.f
     $(FC) -c $(FFLAGS) $(SRC)/sort.f
profil.o: $(SRC)/profil.f
      $(FC) -c $(FFLAGS) $(SRC)/profil.f
polplt.o: $(SRC)/polplt.f $(SRC)/PINDEX.INC
     $(FC) -c $(FFLAGS) $(SRC)/polplt.f
iopol.o: $(SRC)/iopol.f $(SRC)/PINDEX.INC
      (FC) - c (FFLAGS) (SRC) / iopol.f
#pplot.o: $(SRC)/pplot.f $(SRC)/PPLOT.INC
     $(FC) -c $(FFLAGS) $(SRC)/pplot.f
```

- (8) Start Cygwin, and navigate to the location of the XFOIL source code, with appropriate files replaced by typing "cd ../../cygdrive/c/xfoil/src" or to the directory as appropriate.
- (9) At the Cygwin prompt, type "make". The modified executable used for the work conducted in this thesis should compile. The xfoil.exe executable will be built and located in the same directory above in step (8). xfoil.exe may now be relocated as necessary and place in the appropriate directory for MATLAB execution.
- (10) Future modifications may be made to the source code files (*.f) in order to alter the program as further desired. If the source code files are altered, repeat steps (8) and (9) to generate a new executable file.

Appendix H: MATLAB Files for Calculation of Minimum Pressure Envelopes

XBucket.m

```
%Code by Chris Peterson.
%Code intended to produce minimum pressure envelopes using XFOIL to
%calculate minimum pressure for foil geometry. Code can either use NACA 4-
%or 5-digit airfoils built into XFOIL, or may read in properly formatted
%thickness and camber distributions from text files.
clc; clear all; close all;
foil type = 'LOAD'; %Either 'LOAD', or 'NACA' for 4 or 5 digit
if foil_type == 'LOAD'
                           %Filenames thickness & camber, and data file
    foil name = 'foildata';
    load mean = 'Brock08act.txt';
    load thck = 'Brock66act.txt';
elseif foil type == 'NACA'
    foil_name = 'FOUR'; %Or 'FIVE' fo loc = 0.4; %O.X for 4-digt, or 0.05*k for 5-digit (k=1-5)
end
           = '.\xfoil\'; %Specify XFOIL.EXE executable location
xdir
panels = 175; %Sets number of panels if resetting in XFOIL Alpha_lim = [-5 8]; %Angle of attack range %Angle of attack increment
foc rng = [0.00 \ 0.06]; %Camber ratio range (Must be <0.1 for 4-digit
NACA)
foc step = 0.01;
                     %Camber ratio increment
toc_rng = [0.02 0.2];
toc_step = 0.02;
                            %Thicness ratio range
                           %Thickness ratio increment
visc_tog = 0; %1-yes(viscous), 0-no(Inviscid)
iter_lim = 500; %XFOIL viscous calc iteration limit
Re_no = 1e7; %Reynolds number for visc calcs
if visc tog == 1
                           %Add visc functionality to XFOIL command line
    visc_cmd = ['OPER ITER', num2str(iter_lim),'',...
        'OPER VISC ', num2str(Re no), ' '];
elseif visc tog == 0
    visc cmd = '';
%Determine alphas to calculate
A rng = Alpha lim(1):Alpha delta:Alpha lim(2);
%Start of main calculation loops
```

```
j = int8((to c-toc rng(1)+toc step)/toc step);
       %IF below determines if XFOIL database will be used, and creates
       %foil if tabulated data is to be read in
       if strcmp(foil type, 'NACA')
           if strcmp(foil name, 'FOUR')
               name = get\overline{4} nm(fo loc, fo c, to c);
           elseif strcmp(foil name, 'FIVE')
               name = get5 nm(fo loc, to c);
       elseif strcmp(foil type, 'LOAD') %Makes foil if reg'd
           makefoil(to c, fo c, load mean, load thck, foil name);
           name = foil name;
       end
       %CMD generates call to run the XFOIL executable to calc CPmin.
       cmd = [xdir, 'xfoil.exe ',...
           'NORM ',...
           foil type, ' ', name, ' ',...
           'GDES TSET ', num2str(to c), ' ', num2str(fo c), ' ',...
           'GDES X ',...
            visc cmd, ...
           'OPER ALFL ', num2str(Alpha lim(1)),' ',...
           num2str(Alpha delta),' ', num2str(Alpha lim(2)), ' '];
                                          %Calls XFOIL
       system(cmd);
        %Reads in -Cpmin, and x-location of -Cpmin
       fid = fopen('CPMINARRAY.txt');
       clear datain;
       if visc tog == 0;
           datain = textscan(fid, '%f64 %f64', 'headerlines', 1);
       elseif visc tog == 1;
           datain = textscan(fid, '%f64 %f64 %f64', ...
               'headerlines', 1);
       end
       fclose(fid);
       if visc tog == 1;
           cpmni(j,:,k)
                         = datain\{1,1\};
           xcpmni(j,:,k) = datain{1,2};
                         = datain{1,3};
           cpmnv(j,:,k)
                         = datain{1,4};
           xcpmnv(j,:,k)
       elseif visc_tog == 0
           cpmni(j,:,k) = datain{1,1}; %Data array for minimum Cp
           xcpmini(j,:,k) = datain{1,2}; %Data array for location of CPmin
       end
    end
%Generates Bucket diagrams, new plot for each Fo/C
   figure();
   hold on; grid on;
    cmap = colormap(hsv(toc rng(2)/toc step+1)); %Generates color distibution
    set(gca, 'ColorOrder', cmap);
   plot(-cpmni(:,:,k), A_rng(1:length(cpmni))); %Plots Alpha vs. -Cpmin
   xlim([0 3]);
   if fo c > 0
```

```
ylim(Alpha lim);
                                           %Set plot X/Y limits
    else
       ylim([0 8]);
    end
    xlabel('-CP_m_i_n'); ylabel('Angle of Attack (\alpha)');
    if foil type == 'NACA'
        title name = [foil type, ' ', foil name];
    elseif foil type == 'LOAD'
        title name = ['Meanline: ', load mean, '. Thickness: ', load thck];
       title_name = 'UNKNOWN TYPE';
    end
    title({['INVISCID Brockett Diagram', 10, title name, 10,...
        ' Fo/c = ', num2str(fo_c), ' ', '\Delta\alpha = ',
num2str(Alpha delta)]});
    tau = toc rng(1):toc step:toc rng(2);
                                                    %Used for legend
    leg st = cell(1,length(tau));
                                                    %Initializes cells
    for i = 1:length(tau);
                                                   %Set vales to cells
        leg st(i) = {num2str(tau(i))};
    legend(leg st, 'Location', 'SouthEast')
    if visc tog == 1
        figure();
       hold on; grid on;
        cmap = colormap(hsv(toc rng(2)/toc step+1));
                        %Generates color distibution
        set(gca, 'ColorOrder', cmap);
       plot(-cpmnv(:,:,k), A rng(1:length(cpmnv)));
                        %Plots Alpha vs. -Cpmin
       xlim([0 3]);
       ylim(Alpha lim);
                                        %Set plot X/Y limits
       xlabel('-CP m i n'); ylabel('Angle of Attack (\alpha)');
       if foil type == 'NACA'
            title_name = [foil_type, ' ', foil name];
        elseif foil type == 'LOAD'
            title name = ['Meanline: ', load mean, '. Thickness: ',
load thck];
        else
            title name = 'UNKNOWN TYPE';
        end
        title({['VISCOUS Brockett Diagram', 10, title_name, 10,...
            ' Fo/c = ', num2str(fo c), ' \Delta\alpha = ',
num2str(Alpha delta)]});
        tau = toc rng(1):toc step:toc rng(2);
                                                       %Used for legend
                                                       %Initializes cells
        leg st = cell(1,length(tau));
       for i = 1:length(tau);
                                                       %Set vales to cells
            leg st(i) = {num2str(tau(i))};
        legend(leg st, 'Location', 'SouthEast')
    end
end
```

makefoil.m

```
%Code by Chris Peterson. Code will read in specified camber and thickness
   distributions and generate foil geometry file for XFOIL. Thickness and
% camber are scaled to t set and f set.
% Coordinates start at TE, go forward CCW along upper surfact to LE,
% and back to TE along lower surface.
function [] = makefoil(t set, f set, mean type, thick type, save as)
     clc; clear all; close all;
     t_set = 0.1;
              = 0.08;
     f set
     mean type = 'NACAa=08(Brockett).txt';
     thick type = 'NACA66(Brockett).txt';
응
      save as = 'brockett';
              = 'no'; %Generate plot toggle ('yes' or 'no')
make plot
               = 35; %Number of points to make nose parabola. Fails at
N_parab def
numbers < ~20
N parab eval = 11; %Number of points to include at the nose in data
export;
N surf pts
              = 80; %Number of points along body to TE (not including LE)
                       %N parab pts + N surf pts must be < 150
               = 1-2/N parab eval; %Fraction of parabola to use from LE to
fract
0.005.
                       %Max parabola point must be less than 0.005
                       %to prevent sharp cornder at 0.005.
                       %Power for exponential disribution at LE. This
conc fact
              = 2;
                       %concentrates point near tip.
%Get meanline and dy/dx distributions from mean line data base
[x_f fc_o dydx_o] = getmeanline(mean type);
[x t tc o RLE o] = getthickdist(thick type);
%Scale appropriately
                              %uses 1/2 thickness
t set = t set/2;
if max(fc o) \sim = 0
    f scale = f set/max(fc o);
elseif max(fc o) == 0
    f_scale = 0;
end
      = fc o * f scale;
fс
\overline{dy}dx = \overline{dy}dx \circ f scale;
t scale = t set/max(tc o);
tc = tc o * t scale;
      = RLE o * (t scale)^2;
%Find points along RLE nose parabola
x RLE = fract*0.005*(0:1/(N parab def-1):1).^conc fact;
t RLE = sqrt(2*RLE*(x RLE));
%Spline parabola and tabulated data for thickness function
x locs = [x RLE x t(2:end)];
                                               %New combined x/c values
```

```
t fnct = csape(x locs, [1e10 t RLE t c(2:end) 1],[1 0]); %1e8 sets init
slope = \sim inf
%Make x locations for generating data file
    Cosine spacing from 0.005 to TE
x \cos sp = 0.005 + 0.5*0.995*(1-\cos(0:pi/(N surf pts-1):pi));
    %Exponential spacing for nose
x \text{ eval LE} = \text{fract*0.005*(0:1/(N parab eval-1):1).^conc fact};
t eval LE = sqrt(2*RLE*(x eval LE));
x_eval_mb = [x_cos_sp]; %Establishes eval points
t_eval_mb = fnval(t_fnct, x_eval_mb); %Evaluates spline at eval points
x \text{ eval} = [x \text{ eval LE } x \text{ eval mb}];
t eval = [t eval LE t eval mb];
%Spline tabulated data for camber at same x/c locations as thickness
f_f = csape(x_f, f_c);
f eval = fnval(f fnct, x eval);
dydx eval = fnval(fnder(f fnct), x eval);
%Plotting for unrotated parameters
if strcmp(make plot, 'yes')
    figure();
    hold on;
    axis equal;
                             %Set X:Y to unity
    title('Camber, Thickness, and LE Graphical Display')
    xlabel('X/C');
    xlim([-0.01 0.25]);
                            %Set Initial Zoom
        %Plot thickness
        fnplt(t fnct, 'y'); fnplt(f fnct, 'g')
        plot(x t, t_c, 'co'); plot(x_f, f_c, 'ro')
        plot(x RLE, t RLE, 'k.');
        %Plot RLE Circle and parabola for viewing on plot
        plot(RLE - RLE*cos(0:pi/100:pi), RLE*(sin(0:pi/100:pi)), 'b:');
        plot((0:1/10000:0.2), sqrt(2*RLE*(0:1/10000:0.2)), 'r:');
        %Plot camber
    legend('Splined Thickness', 'Splined Camber',...
         'Tabulated Thickness (Scaled)', 'Tabulated Camber (Scaled)',...
         'Calcuated Parabola', 'Leading Edge Radius', 'LE Parabola',...
        'Location', 'southeast')
end
%Calculate upper and lower surface ordinates
x u = x eval - t eval.*sin(atan(dydx eval));
y u = f eval + t eval.*cos(atan(dydx eval));
x l = x eval + t eval.*sin(atan(dydx eval));
y l = f eval - t eval.*cos(atan(dydx eval));
%Solve for most forward point on foil
[x \text{ fwd, min i}] = \min(x \text{ u});
y \text{ fwd} = y \text{ u(min i)};
%New plot for actual upper and lower surfaces
if strcmp(make plot, 'yes')
    figure();
```

```
hold on;
    axis equal;
                           %Set X:Y to unity
    xlim([0 1]); %Set Initial Zoom
    plot(x_u, y_u, 'b-', x_u, y_u, 'r.')
    plot(x_1, y_1, 'b-', x_1, y_1, 'r.');
    plot(x eval, f eval, 'g-', x eval, f eval, 'r.')
    plot(x fwd, y fwd, 'kp')
end
%Combine coordinates into a single array of points from TE along upper
%surface around LE back to TE along lower surface
x comb = [fliplr(x u) x l];
y comb = [fliplr(y u) y l];
%Rotate and scale such that max forward point is at 0,0, and TE is at 0,1.
%Assumes TE is already at 0,0 (Uses method in Brockett Report)
shift ang = atan(y fwd/(1-x fwd));
%Scaled chord length back to 1 (accounts for portion forward of 0)
x \text{ scaled} = (x \text{ comb-}x \text{ fwd})./(1-x \text{ fwd});
y scaled = (y comb-y fwd)./(1-x fwd);
%Rotate so that most forward point is at 0,0
x rot = (x scaled.*cos(shift_ang) - y_scaled.*sin(shift_ang))/...
           sqrt(1+(y_fwd/(1-x_fwd))^2);
y rot = (y scaled.*cos(shift ang) + x scaled.*sin(shift ang))/...
           sqrt(1+(y fwd/(1-x fwd))^2);
%New plot for final upper and lower surfaces
if strcmp(make plot, 'yes')
    figure();
    hold on;
    title('Final Points exported to Data File.');
    axis equal;
                           %Set X:Y to unity
    plot(x rot, y rot, x rot, y rot, 'r.');
    legend('Connect the dots', 'Actual data points');
%Write to text file for use in XFOIL.
                               %save as is file name to be written to
cmd = ['del ', save as];
system(cmd);
                                %Delets previous file
fid = fopen(save as, 'w');
for i = 1:length(x rot)
    fprintf(fid, '%10.8f %10.8f\n', x rot(i), y rot(i));
fclose(fid);
```

getmeanline.m

```
Code by Chris Peterson
% Code developed to read meanline information from data file 'filename'.
% Data will be read in from file, and returned to function call. Data
% return is vectors containing x-locations, camber distribution, and
   camber line slope values. Function checks for 999 value specifying
   less data points than standard input format.
function [x loc f c dy dx] = getmeanline(filename)
cd('./Meanline');
input = dlmread(filename, '\t', 4, 0);
       = input';
x loc in= M(1,:)/100;
f c in = M(2,:)/100;
dy dx in= M(3,:);
for i=1:length(x loc in)
    if x \log in(i) = 9.99 %Checks to see if formatted with less points
       \overline{x} loc = x loc in(1:i-1);
             = f_c_in(1:i-1);
        fc
       dy dx = dy dx in(1:i-1);
       cd ..;
       return
    else
       x loc = x loc in;
       fc = fcin;
       dy_dx = dy_dx in;
    end
end
cd ..
```

getthickdist.m

```
Code by Chris Peterson
% Code developed to read thickness information from data file 'filename'.
% Data will be read in from file, and returned to function call. Data
% return is vectors containing x-locations, thickness distribution, and
   value of leading edge radius. Function checks for 999 value specifying
   less data points than standard input format.
function [x loc t c RLE] = getthickdist(filename)
cd('./Thickness');
input = dlmread(filename, '\t', [4 0 29 2]);
       = input';
x loc in= M(1,:)/100;
t c in = M(2,:)/100;
fid
      = fopen(filename);
RLE
      = textscan(fid, '%s', 'headerlines', 29);
fclose all;
RLE = str2num(RLE{1}{7})/100;
for i=1:length(x loc in)
    if x loc in(\overline{i}) == 9.99 %Checks to see if formatted with less points
        \overline{x} loc = x loc in(1:i-1);
                = t^c in(1:i-1);
        t c
        cd ..;
        return
    else
        x loc = x loc in;
              = t c in;
        t c
    end
end
cd ..;
```

get4 nm

```
%Code by Chris Peterson. Code generates a 4 digit string based on location %of max camber, camber and thickness to generate NACA 4-digit designation.
```

```
function [name] = get4_nm(loc, fo_c, to_c)

if fo_c == 0
    no1 = '0';
    no2 = '0';

else
    no1 = num2str(int8(100*fo_c));
    no2 = num2str(int8(10*loc));

end

if to_c < 0.1
    no34 = strcat('0', num2str(int8(100*to_c)));

else
    no34 = num2str(int8(100*to_c));
end

name = strcat(no1, no2, no34);</pre>
```

get5 nm.m

 $Code\ by\ Chris\ Peterson.$ Code generates a 5 digit string based on location for max camber, camber and thickness to generate NACA 4-digit designation.

Appendix I: Meanline and Camber Data File Format

Meanline and thickness distributions for various NACA foils are available at the Public Domain Aeronautical Software website (http://www.pdas.com/avd.htm). The formats of these files were used as input for the meanline and thickness distributions, and the MATAB code assumes similar formatting for other tabulated offsets. Examples of meanline and thickness data files are shown below for the NACA a=0.3 Meanline, and the NACA 66-008 thickness distributions:

NACAa=0.8.txt				NACA66-008.txt			
NACA Mean Line a=0.3			NACA 66-008				
(Stations and ordinates given			(Stations and ordinates given				
in per cent of airfoil chord)			in per cent of airfoil chord)				
X	у	dy/dx	X	у	dy/dx		
0	0	0	0	0	0		
0.5	0.3892	0.6554	0.5	0.6111	0.5674		
0.75	0.5463	0.6052	0.75	0.7341	0.4353		
1.25	0.8317	0.5416	1.25	0.9151	0.306		
2.5	1.4478	0.454	2.5	1.2183	0.2079		
5	2.4575	0.3634	5	1.6716	0.1605		
7.5	3.2925	0.3078	7.5	2.0321	0.1303		
10	4.008	0.2662	10	2.3336	0.1123		
15	5.1721	0.2025	15	2.8245	0.0857		
20	6.052	0.1507	20	3.2003	0.0658		
25	6.6853	0.1028	25	3.4904	0.0505		
30	7.0721	0.0483	30	3.7091	0.0372		
35	7.1754	-0.002	35	3.8642	0.0253		
40	7.0738	-0.0371	40	3.9603	0.0131		
45	6.8162	-0.0649	45	3.9984	0.0016		
50	6.4333	-0.0875	50	3.9777	-0.0102		
55	5.9488	-0.1057	55	3.8945	-0.0236		
60	5.3828	-0.1201	60	3.7378	-0.0408		
65	4.7531	-0.1312	65	3.4659	-0.0693		
70	4.0763	-0.139	70	3.0593	-0.0915		
75	3.3683	-0.1436	75	2.5713	-0.1039		
80	2.6453	-0.145	80	2.0256	-0.1137		
85	1.9243	-0.1428	85	1.445	-0.1169		
90	1.2244	-0.1364	90	0.8674	-0.1131		
95	0.5698	-0.1243	95	0.3378	-0.0952		
100	0	0	100	0	-0.0038		
			L.E. r	L.E. radius = 0.389 percent chord			
	F						

All values are specified as a percentage of chord length. Note that for meanline data, 'y' values represent camber offsets, and dy/dx is camberline slope. For thickness distribution, 'y' represents thickness values perpendicular to the meanline, and dy/dx is thickness slope. In addition, leading edge radius must be specified.

If desired, meanline and thickness for arbitrary foil shapes may be specified using the above format, or alternatively, if offsets are available, but not at the locations specified above, the following formats may also be used. These meanline and thickness offsets were taken from reference [4]

Brock08act.	<u>txt</u>	Brock66act.txt				
NACA Mean	\overline{L} ine a=0.8(mo	NACA 66 (Mod)-From Brockett				
(Stations and	ordinates giver	(Stations and ordinates given				
in per cent of	f airfoil chord)	in per cent of airfoil chord)				
X	y	dy/dx	X	y	dydx	
0	0	0.71485	0	0	0	
0.7596	0.6006	0.66001	0.7596	0.817	0	
3.0154	1.8381	0.47712	3.0154	1.608	0	
6.6987	3.3684	0.36751	6.6987	2.388	0	
11.6978	4.9874	0.28681	11.6978	3.135	0	
17.8606	6.5407	0.22096	17.8606	3.807	0	
25	7.9051	0.1635	25	4.363	0	
32.899	8.9831	0.11071	32.899	4.76	0	
41.3176	9.6994	0.06001	41.3176	4.972	0	
50	10	0.00914	50	4.962	0	
58.6824	9.8503	-0.04448	58.6824	4.712	0	
67.101	9.2306	-0.10483	67.101	4.247	0	
75	8.1212	-0.18132	75	3.612	0	
82.1394	6.3884	-0.31892	82.1394	2.872	0	
88.3022	4.2227	-0.37243	88.3022	2.108	0	
93.3013	2.3423	-0.37425	93.3013	1.402	0	
96.9846	0.9982	-0.35148	96.9846	0.83	0	
99.2404	0.2365	-0.32028	99.2404	0.462	0	
100	0	-0.30025	100	0.333	0	
999	0	0	999	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	
			L.E. radius = .448 percent chord			

The file formats for Brock08act.txt and Brock66act.txt utilize the same number of rows and columns as the previous formats (NACAa=0.8.txt and NACA66-008.txt). The only difference is that when there are less than 26 offset locations, the number 999 must be put after the last data point. This instructs the code to stop reading in data points. All other values after the last offset location must be filled in with zeros to maintain proper file format and size. Also, dy/dx values for thickness distributions are not required, and may be filled in with zeros if unknown.